Exploring the physics behind the interaction between acoustic waves and unsaturated porous media

Wei-Cheng Lo

Hydraulic and Ocean Engineering, National Cheng Kung University, Taiwan

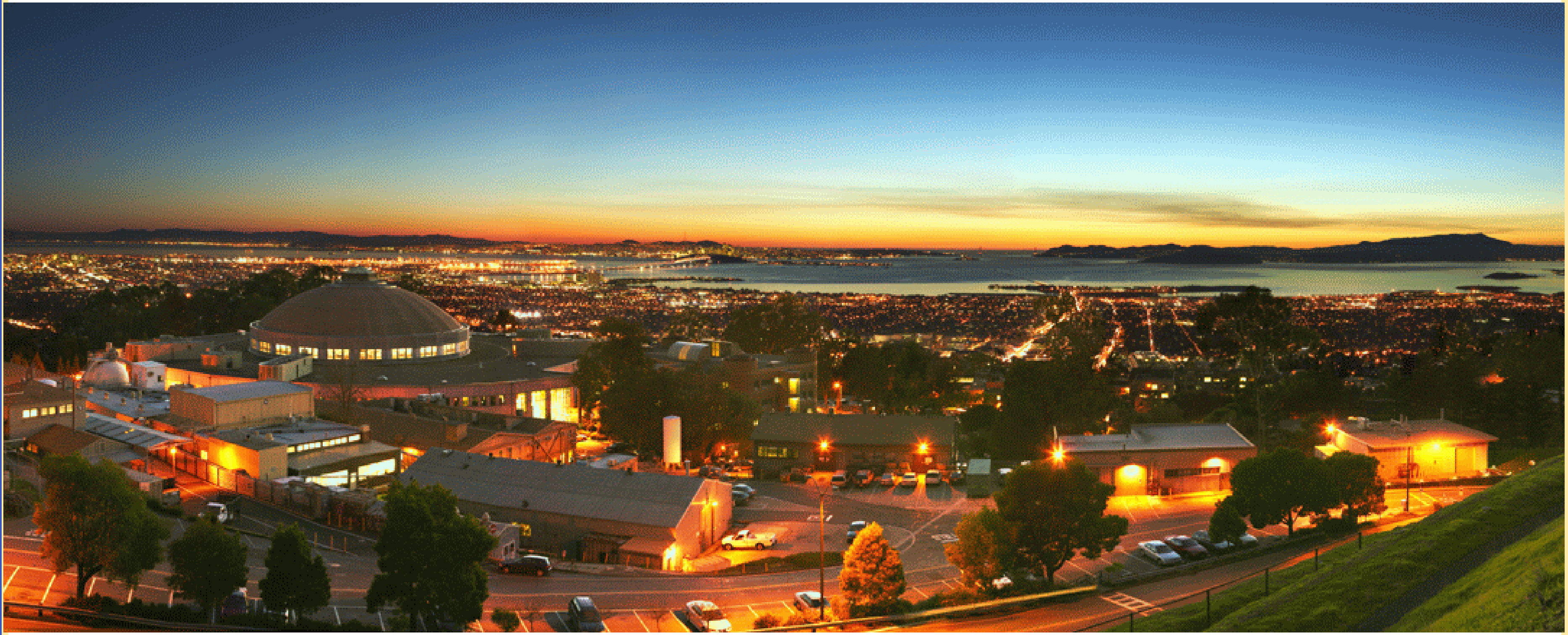
Garrison Sposito

Civil and Environmental Engineering, University of California at Berkeley

Ernest L. Majer

Geophysics, Earth Sciences Division, Lawrence Berkeley National Laboratory

Funding: Department of Energy, USA



+26 kg

Berkeley, 2003

Cornell, 1994

Compensation at University of California (2012)

- Berkeley

  Gross pay above $200K – 430 people!

  Gross pay above $300K – 89 people!

- Los Angles

  Gross pay above $200K – 1471 people!

  Gross pay above $300K – 611 people!

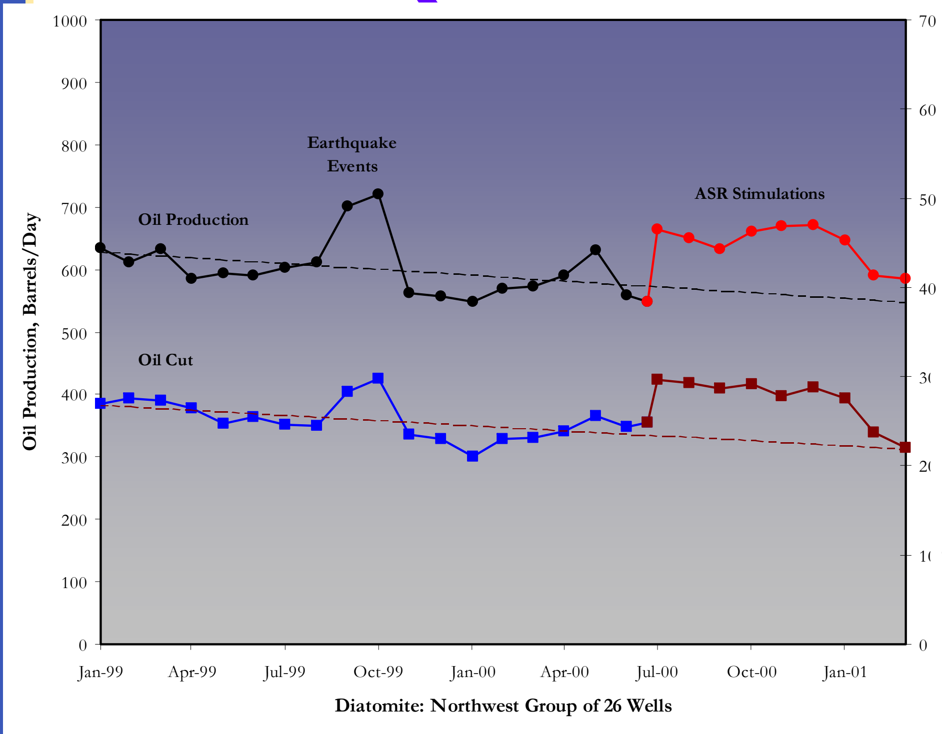
$ 305,733.30 !! => NT $25128/per day

https://ucannualwage.ucop.edu/wage/

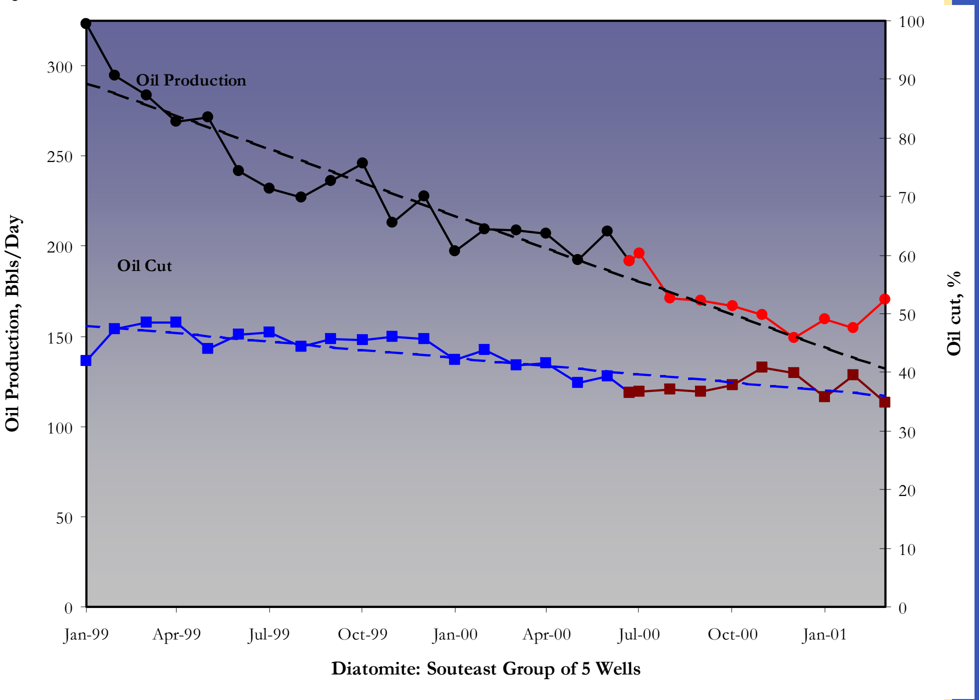
Robert E. Horton Medal

- 2013 Soroosh Sorooshian (UC Irvine) 2012 Keith Beven (Lancaster)
- 2011 Murugesu Sivapalan (UIUC) 2010 Jacob Bear (IIT)
- 2009 William E. Dietrich (UC Berkeley) 2008 Vijay K. Gupta (UCo, Boulder)
- 2007 Rafael L. Bras (GIT) 2006 Thomas Schmugge (USDA)
- 2005 Gedeon Dagan (Tel Aviv) 2004 Garrison Sposito (UC Berkeley)
- 2003 Shlomo P. Neuman (U Arizona) 2002 Jean-Yves Parlange (Cornell)
- 2001 Donald R. Nielsen (UC Davis) 2000 M. Gordon Wolman (Die)
- 1999 Wilfried H. Brutsaert (Cornell) 1998 Ignacio Rodriguez-Iturbe (Princeto
- 1997 John D. Bredehoeft 1996 Mark Meier
- 1995 Don Kirkham 1994 Mikhail I. Budyko
- 1992 Luna B. Leopold 1990 Paul A. Witherspoon
- 1988 Peter S. Eagleson 1986 Abel Wolman
- 1984 Charles V. Theis 1982 John R. Philip
- 1980 William C. Ackermann 1978 Harold A. Thomas, Jr.
- 1976 Walter B. Langbein

Motivation (Field Observation)

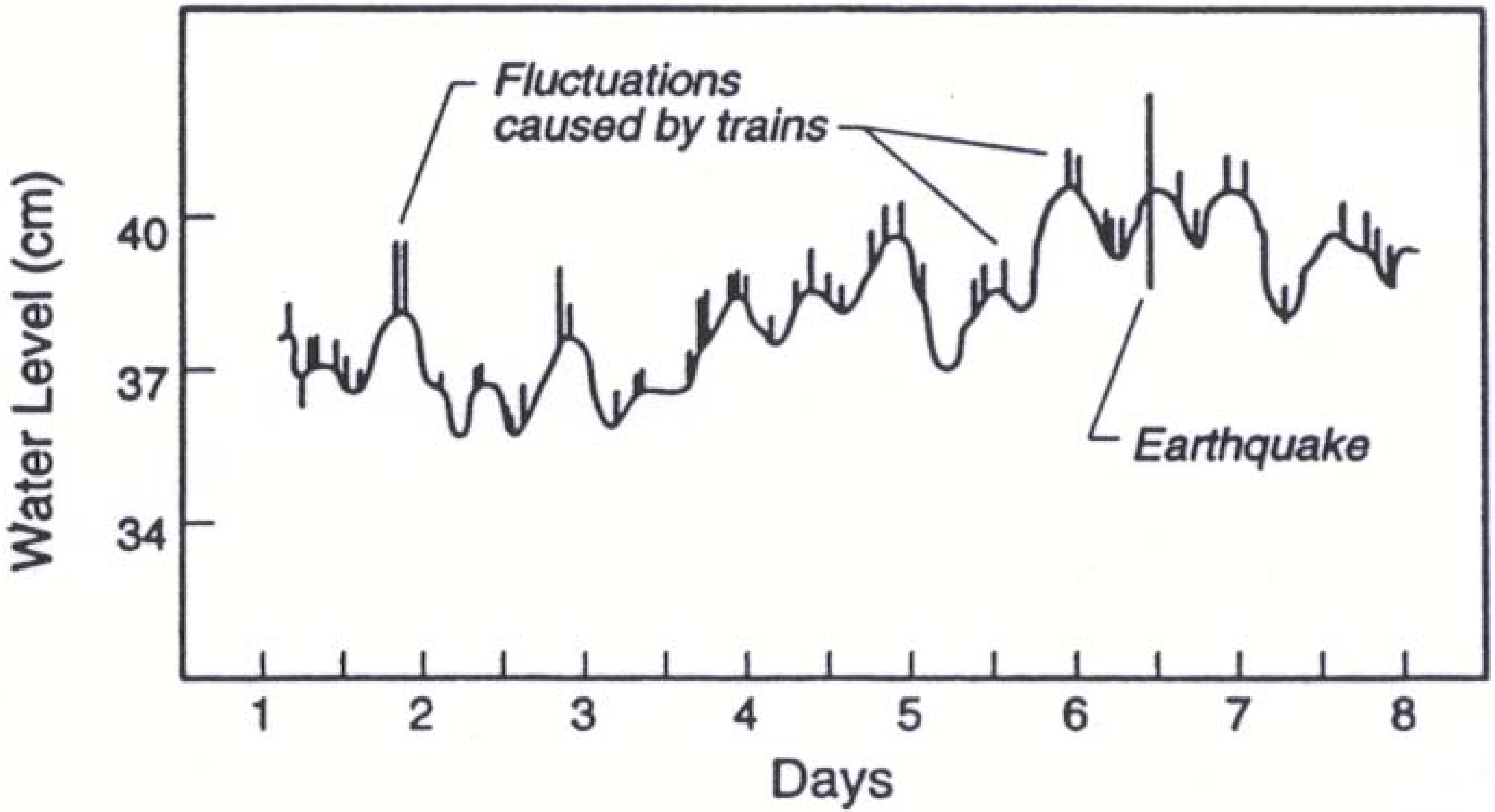


Group of 26 Wells to the Northeast that responded both to the earthquake events of Sept-Oct 1999 and to ISS



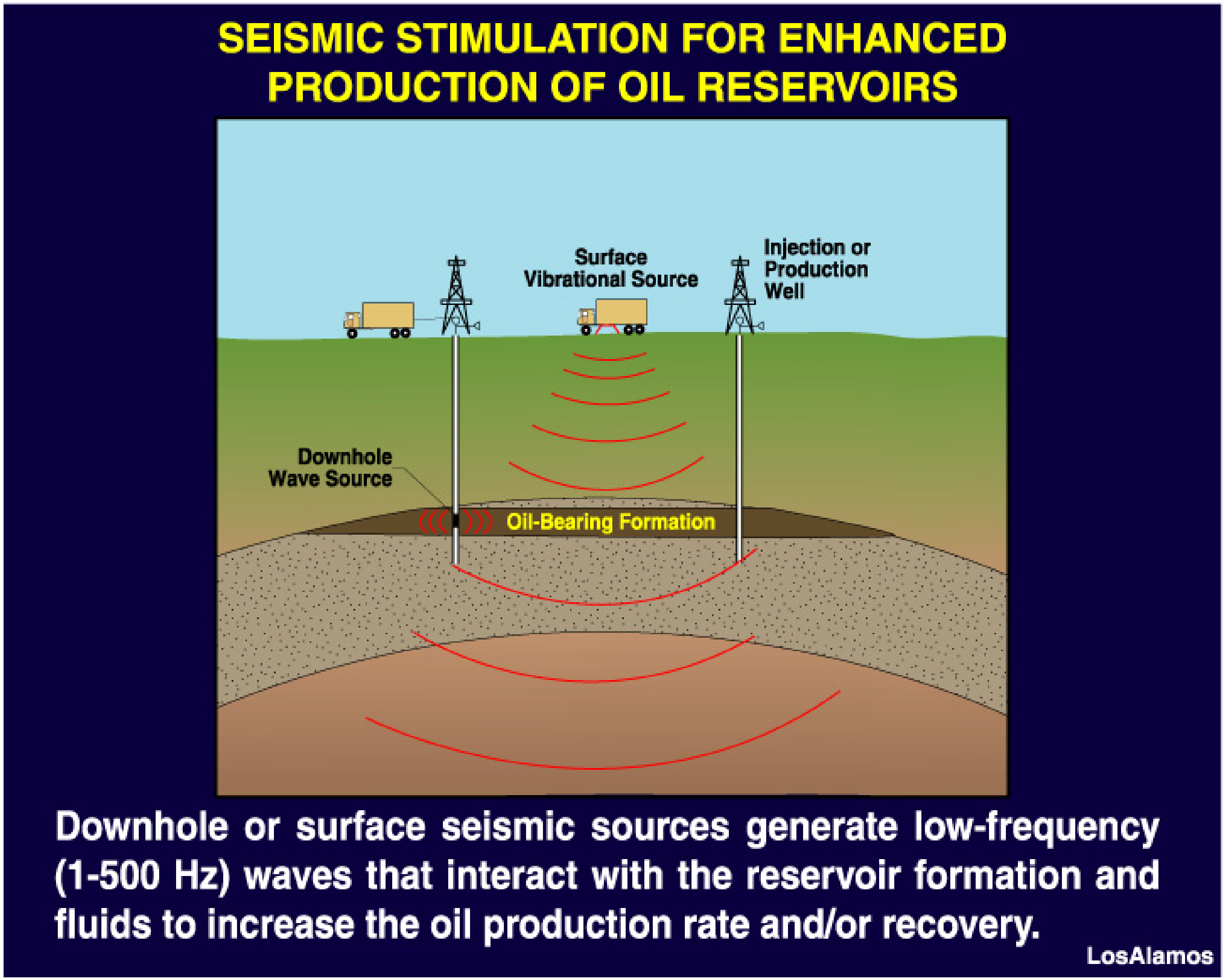
Group of 5 Wells to the Southeast that did not respond either to the earthquake events or to ISS

Field Observation

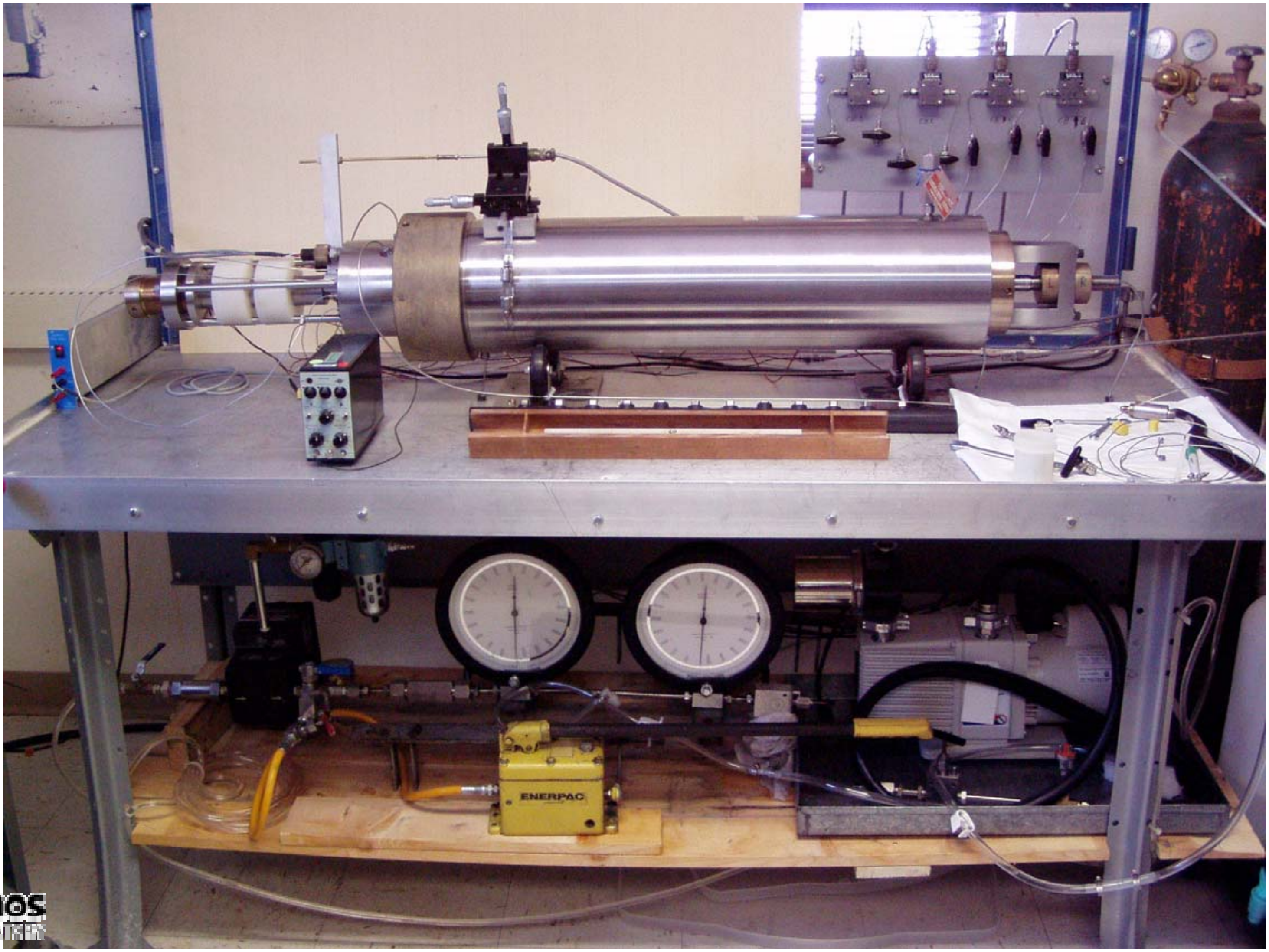


Fluctuations of water level in a 52-m deep well induced by seismic waves excited by passing trains and an earthquake.

Field Experiments

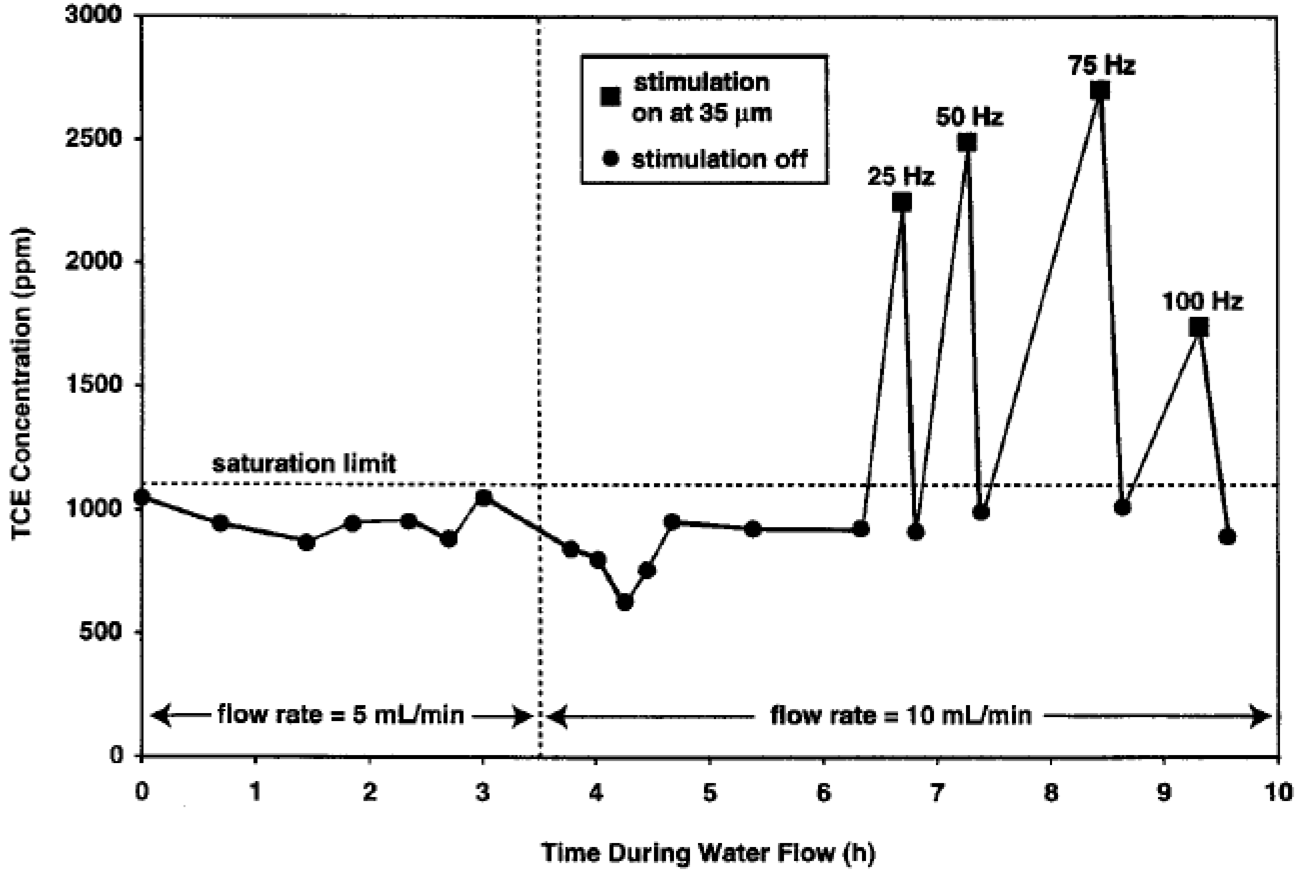


Laboratory Experiment



Los Alamos
NATIONAL LABORATORY

Laboratory Observation



Stimulation time: 360s

Pore pressure gradient: 3 kPa/m

Free-phase TCE observed

Permeability: 1.1x10^{-10} m^2 (111 d)

Roberts et al., *Environ. Engin. Sci.* **18**(2):67-79 (2001)

Problems to be Addressed

- **Although the potential benefits of seismic wave stimulation have been demonstrated in laboratory and field experiments, they lack a *sound theoretical basis*.**
- **Seismic wave stimulation will be not fully developed into a predictable and reliable field technology until more fundamental research is performed to understand better the basic science controlling enhancement phenomena.**

Statement of Problem

- Porous medium containing *two* immiscible fluids (oil and water or air and water)
- Solid: porous, isotropic, homogeneous, and elastic
- Fluids: compressible and viscous

Methodology

Two-phase fluid flow in porous media

[*continuum mechanics of mixtures*]

Coupled

Elastic wave propagation

[*linear stress-strain relations*]

Classical Hydrological Model

- An *uncouple* model - representing the intricate interaction between interstitial fluid flow and solid matrix deformation simply using a single lumped parameter, known as storage coefficient.

Classical Hydrological Model

Transient groundwater flow in confined aquifer

$$\frac{T}{S}\nabla^2 h = \frac{\partial h}{\partial t}$$

$$S = b\rho g(\alpha + n\beta)$$

Terzaghi theory

$$\alpha\frac{\partial^2 p}{\partial z^2} = \frac{\partial p}{\partial t} \qquad \alpha = \frac{K_{sat}}{\rho g m_v}$$

Physically-based Model (Poroelasticity)

- **The solid and fluid constituents should be treated on an equal footing, and their displacement vectors were systemically formulated in the *coupled* equations of motion.**

Mass Balance Equations

$$\frac{\partial(\rho_\alpha \theta_\alpha)}{\partial t} + \vec{\nabla}\cdot(\rho_\alpha \theta_\alpha \vec{v}_\alpha) = 0$$

Storage Outflow

Momentum Balance Equations

$$\rho_\alpha \theta_\alpha \frac{D^\alpha \vec{v}_\alpha}{Dt} - \vec{\nabla} \cdot \overline{\overline{t}}_\alpha - \rho_\alpha \theta_\alpha \vec{g} - \rho_\alpha \theta_\alpha \vec{M}_\alpha = \vec{0}$$

Inertia | Stress | Gravity | Interphase Exchange

Constraints on Constitutive Relationships

- **Objectivity**
- **Symmetry**
- **Entropy inequality**
- ***Linearity***

Mass Balance Equations with Constitutive Relationships

$$\frac{\partial(\rho_\alpha \theta_\alpha)}{\partial t} + \vec{\nabla} \cdot (\rho_\alpha \theta_\alpha \vec{v}_\alpha) = 0$$

no change!

Momentum Balance Equations with Constitutive Relationships

Fluid

$$\rho_\alpha \theta_\alpha \frac{D^\alpha \vec{v}_\alpha}{Dt} = -\theta_\alpha \vec{\nabla} p_\alpha + \rho_\alpha \theta_\alpha \vec{g} + R_{\alpha\alpha} \cdot (\vec{v}_\alpha - \vec{v}_s) + \sum_\beta A_{\alpha\beta} \cdot (\vec{a}_\beta - \vec{a}_s) \qquad \alpha = 1,2 \quad \beta = 1,2$$

Solid

$$\rho_s \theta_s \frac{D^s \vec{v}_s}{Dt} = \vec{\nabla} \cdot \bar{\bar{t}}_s - p_1 \vec{\nabla} \theta_1 - p_2 \vec{\nabla} \theta_2 + \rho_s \theta_s \vec{g} - \sum_\alpha R_{\alpha\alpha} \cdot (\vec{v}_\alpha - \vec{v}_s) - \sum_\alpha \sum_\beta A_{\alpha\beta} \cdot (\vec{a}_\beta - \vec{a}_s)$$

Linear Stress-Strain Relations in Unsaturated Porous Media

$$-\phi S_1 p_1 = a_{12}\vec{\nabla}\cdot\vec{u}_s + a_{22}\vec{\nabla}\cdot\vec{u}_1 + a_{23}\vec{\nabla}\cdot\vec{u}_2,$$

$$-\phi(1-S_1)p_2 = a_{13}\vec{\nabla}\cdot\vec{u}_s + a_{23}\vec{\nabla}\cdot\vec{u}_1 + a_{33}\vec{\nabla}\cdot\vec{u}_2,$$

$$\bar{\bar{t}}_s = 2G\bar{\bar{e}} + [(a_{11}-\frac{2}{3}G)\vec{\nabla}\cdot\vec{u}_s + a_{12}\vec{\nabla}\cdot\vec{u}_1 + a_{13}\vec{\nabla}\cdot\vec{u}_2]\bar{\bar{\delta}}$$

where $\vec{u}_\alpha$ = displacement of the α phase

a_{ij} = elastic coefficients

Porous Medium with *Two Fluids* (Acoustic Motions)

Governing Equations [Lo *et al.*, 2005, *Water Resources Researches*]:

$$\rho_s\theta_s\frac{\partial^2 e}{\partial t^2}+A_{11}\left(\frac{\partial^2\varepsilon_1}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)+A_{12}\left(\frac{\partial^2\varepsilon_2}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)+A_{21}\left(\frac{\partial^2\varepsilon_1}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)$$

$$+A_{22}\left(\frac{\partial^2\varepsilon_2}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)+R_{11}\left(\frac{\partial\varepsilon_1}{\partial t}-\frac{\partial e}{\partial t}\right)+R_{22}\left(\frac{\partial\varepsilon_2}{\partial t}-\frac{\partial e}{\partial t}\right)$$

$$=a_{11}\nabla^2 e+a_{12}\nabla^2\varepsilon_1+a_{13}\nabla^2\varepsilon_2$$

$$\rho_1\theta_1\frac{\partial^2\varepsilon_1}{\partial t^2}-A_{11}\left(\frac{\partial^2\varepsilon_1}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)-A_{12}\left(\frac{\partial^2\varepsilon_2}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)-R_{11}\left(\frac{\partial\varepsilon_1}{\partial t}-\frac{\partial e}{\partial t}\right)$$

$$=a_{21}\nabla^2 e+a_{22}\nabla^2\varepsilon_1+a_{23}\nabla^2\varepsilon_2$$

$$\rho_2\theta_2\frac{\partial^2\varepsilon_2}{\partial t^2}-A_{21}\left(\frac{\partial^2\varepsilon_1}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)-A_{22}\left(\frac{\partial^2\varepsilon_2}{\partial t^2}-\frac{\partial^2 e}{\partial t^2}\right)-R_{22}\left(\frac{\partial\varepsilon_2}{\partial t}-\frac{\partial e}{\partial t}\right)$$

$$=a_{31}\nabla^2 e+a_{32}\nabla^2\varepsilon_1+a_{33}\nabla^2\varepsilon_2$$

Porous Medium with *One Fluid* (Acoustic Motions)

Governing Equations [Biot, 1956]:

$$\frac{\partial^2}{\partial t^2}(\rho_{11}e+\rho_{12}\varepsilon)+b\frac{\partial}{\partial t}(e-\varepsilon)=\nabla^2(Pe+Q\varepsilon)$$

$$\frac{\partial^2}{\partial t^2}(\rho_{12}e+\rho_{22}\varepsilon)-b\frac{\partial}{\partial t}(e-\varepsilon)=\nabla^2(Qe+R\varepsilon)$$

where $e=\vec{\nabla}\cdot\vec{u}_s=$ dilatation of solid, $\varepsilon=\vec{\nabla}\cdot\vec{u}_f=$ dilatation of fluid

$\vec{u}_s$ and $\vec{u}_f$ are the displacement vectors of solid and fluid

$(P-\frac{4}{3}G)K_s^{-1}+QK_f^{-1}=1-\phi$ and $QK_s^{-1}+RK_f^{-1}=\phi$ are elasticity parameters

b is the Biot viscous coupling parameter (inversely proportional to permeability)

ρ_{11}, ρ_{12}, and ρ_{22} ($\rho_{12}<0$) are the Biot inertial coupling parameters

Dispersion Relations

$$D_{11}\left(\frac{\omega^2}{k^2}\right)^3 + D_{22}\left(\frac{\omega^2}{k^2}\right)^2 + D_{33}\left(\frac{\omega^2}{k^2}\right) + D_{44} = 0$$

This equation is a cubic polynomial in $\frac{\omega^2}{k^2}$ and, therefore,

it will in general have three complex roots.

When the wave excitation frequency is stipulated, the corresponding phase speed and attenuation coefficient can be deduced.

The amplitude of the bulk waves always diminishes with distance, and this condition requires $k_i > 0$, which, in turn, implies that only three of the six solutions for the attenuation coefficient are physically possible.

$k = k_r + k_i$ where $k_r =$ wave number $k_i =$ attenuation

Dispersion Relations for the Free Vibration Problem

Input elasticity and hydraulic data

Viscous and inertial coupling parameters
Water retention curve
Hydraulic conductivity function

$e = \beta_s \exp i(kz - \omega t)$

$\varepsilon_1 = \beta_1 \exp i(kz - \omega t)$

$\varepsilon_2 = \beta_2 \exp i(kz - \omega t)$

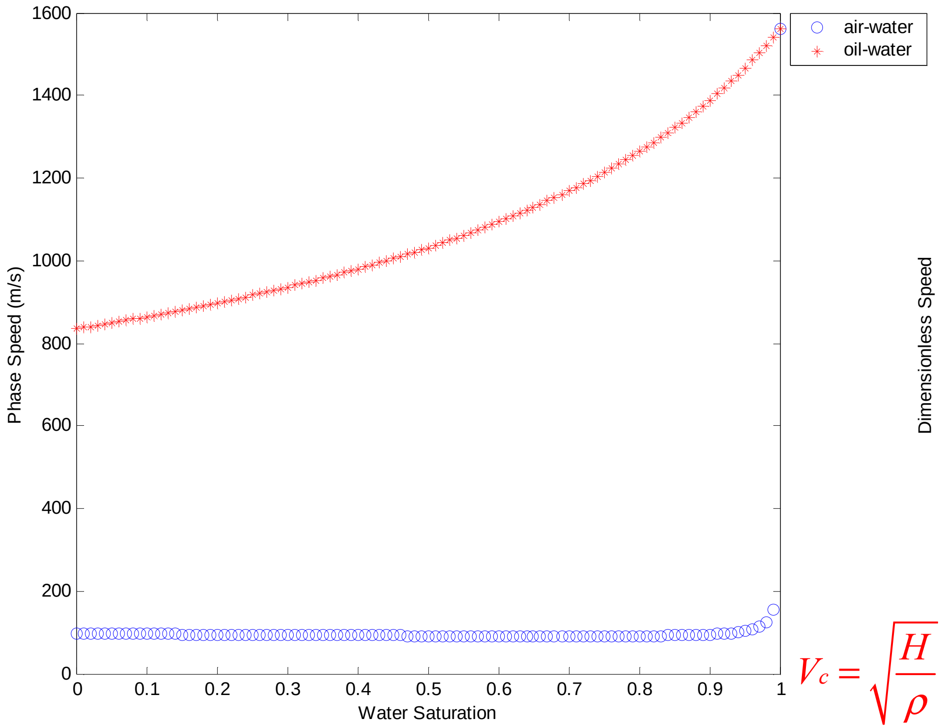
Select vibrational frequency

Three roots for wave number

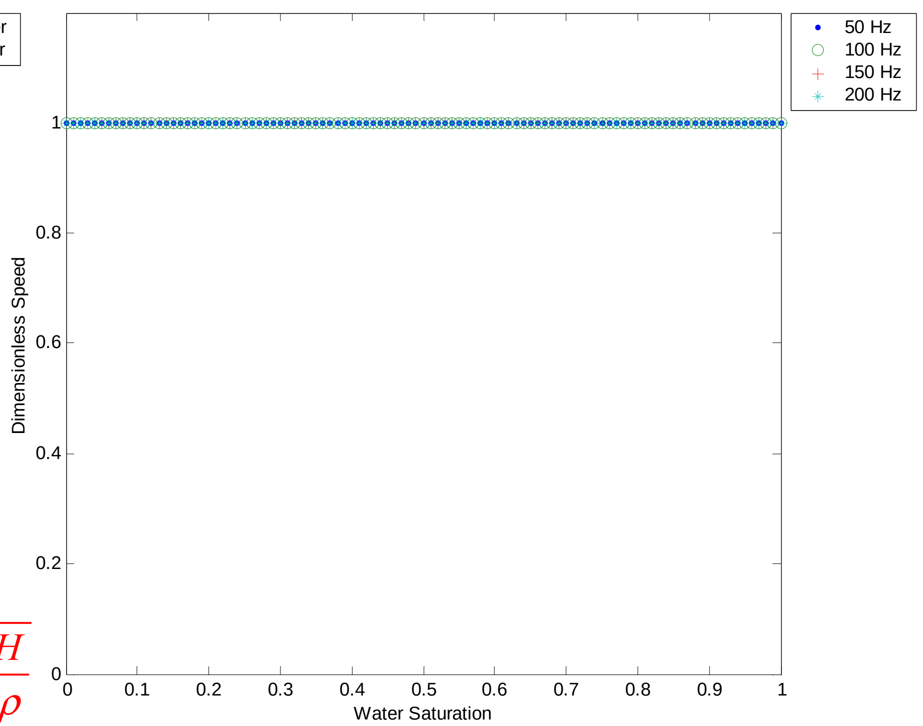
$\bar{v} = \dfrac{\omega}{k_r}$ (Phase Velocity)

$k_i = \mathrm{Im}(k)$ (Attenuation Coefficient)

Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Phase Velocity (P1 Wave)



$$V_c = \sqrt{\frac{H}{\rho}}$$

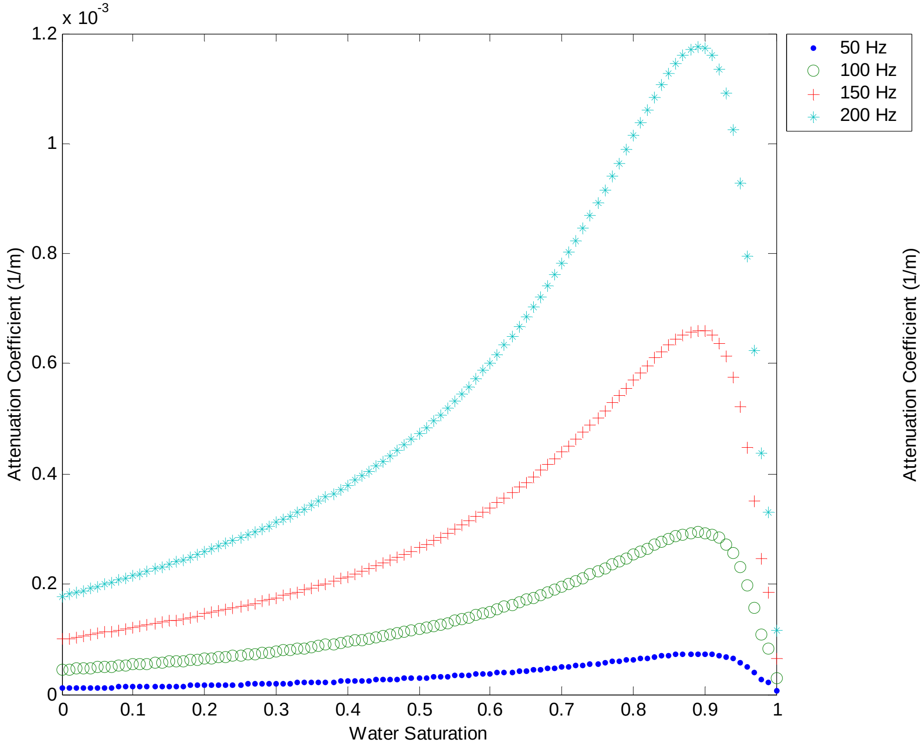


where $H = a_{11} + a_{22} + a_{33} + 2(a_{12} + a_{13} + a_{23}) + \frac{4}{3}G$

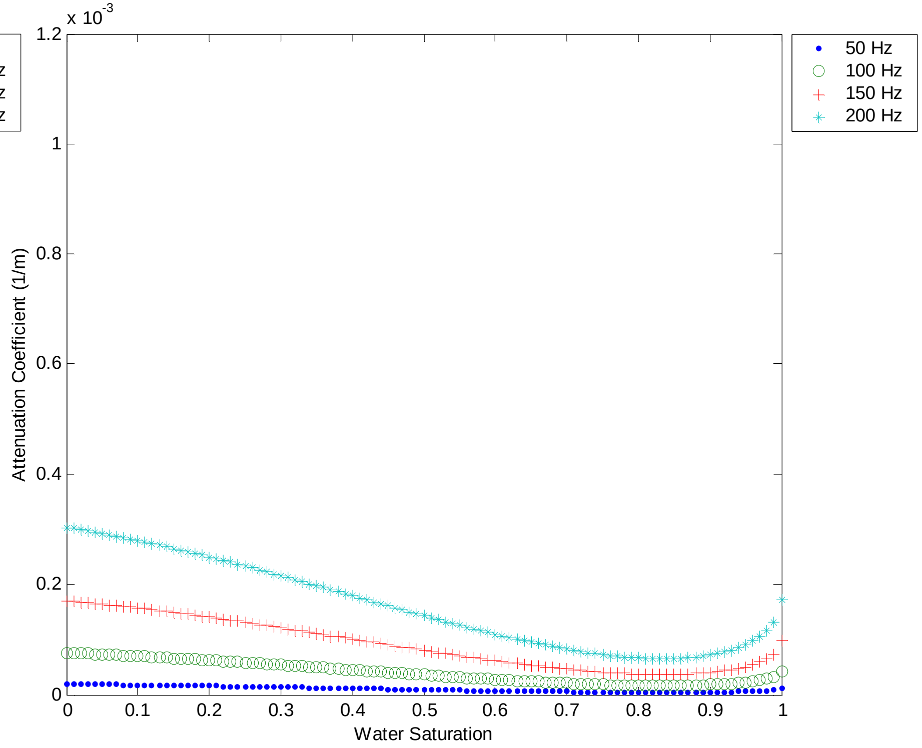
$$\rho = \sum \rho_\alpha \theta_\alpha$$

Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Attenuation Coefficient (P1 Wave)

air-water system



oil-water system



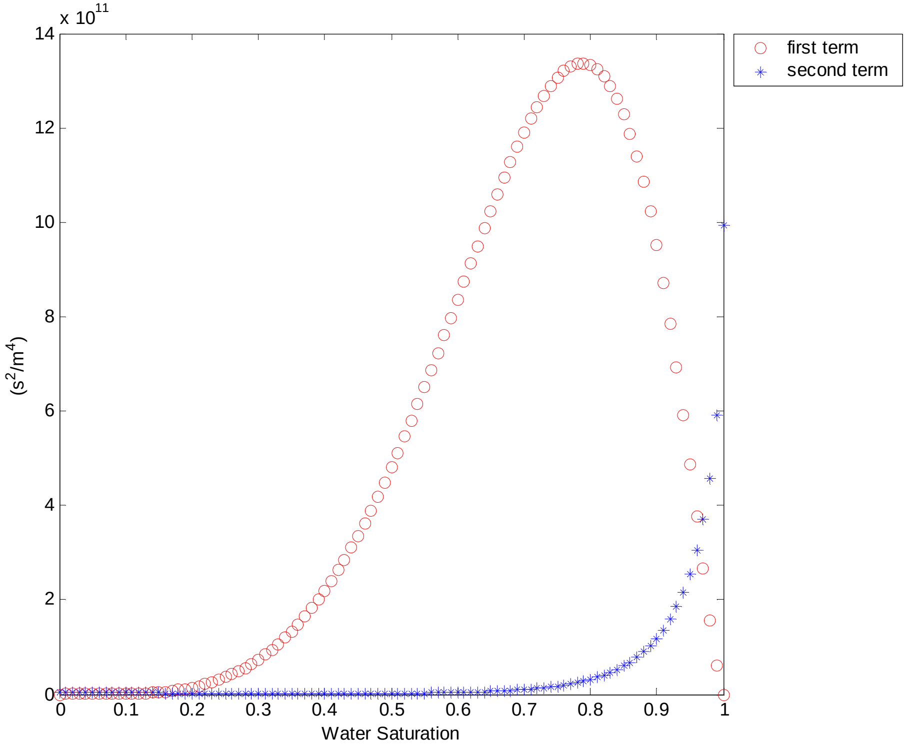
teamed by NCKU, UC Berkeley, and LBNL

Physical Mechanism

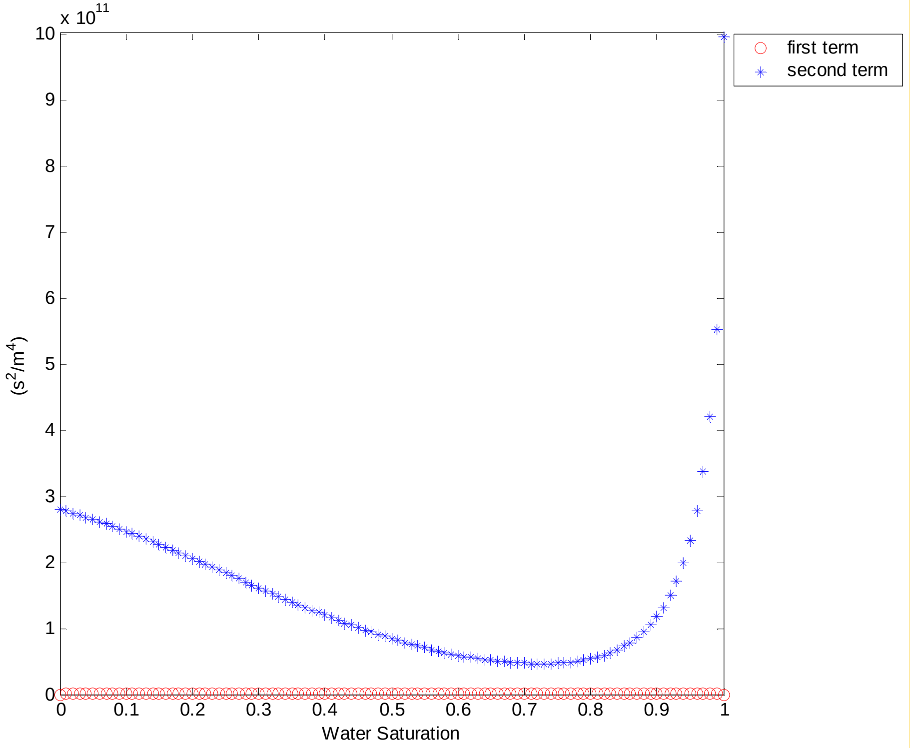
- First term is proportional to the square of the difference in material densities of the two pore fluids, multiplied by the product of their relative mobilities.
- A second term in the model expression is inversely proportional to the square of an average kinematic shear viscosity weighted by relative permeability.
- The first term should be large for an air-water mixture, but small for an oil-water mixture, whereas the reverse should be true for the second term.

Physical Mechanism

air-water system

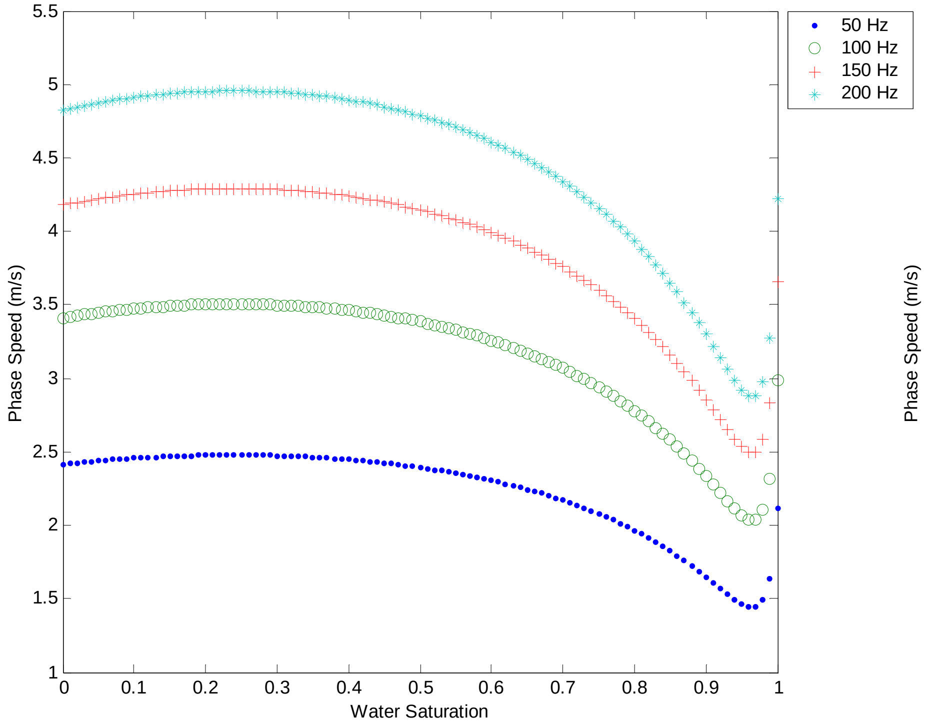


oil-water system

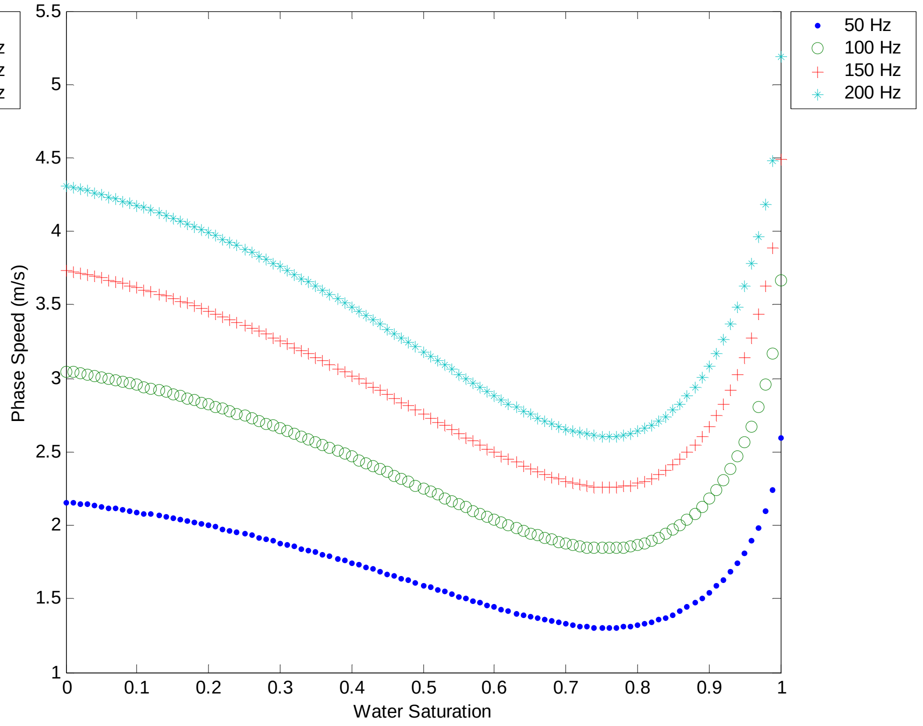


Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Phase Velocity (P2 Wave)

air-water system

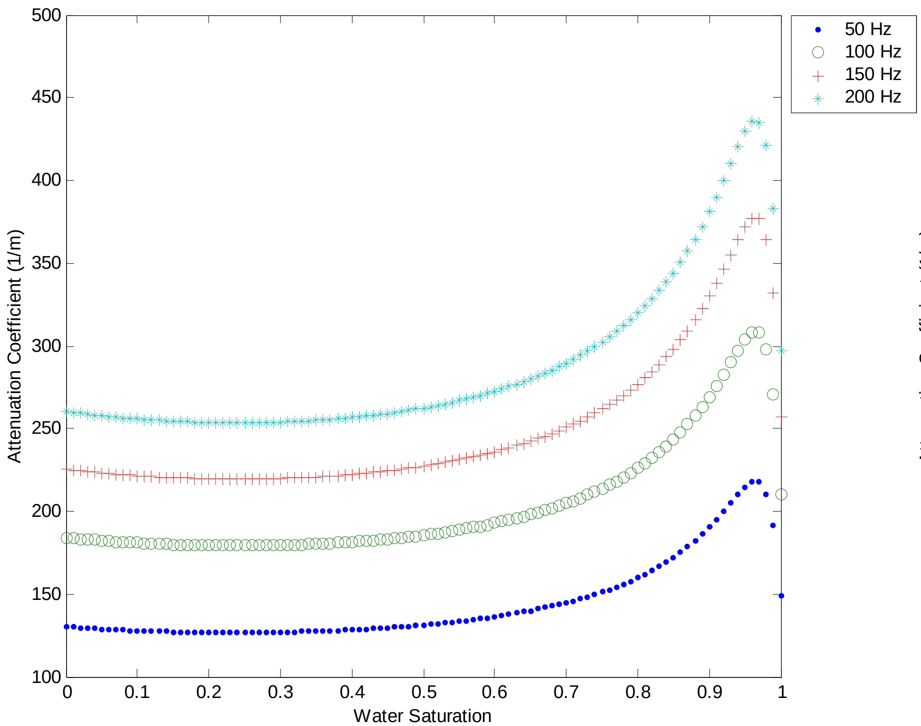


oil-water system

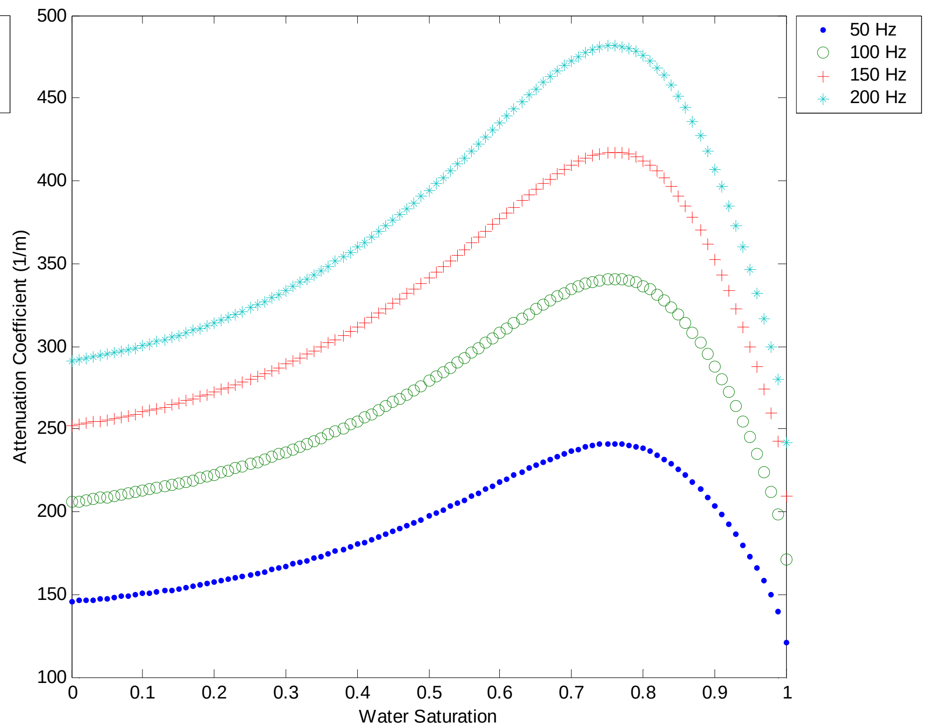


Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Attenuation Coefficient (P2 Wave)

air-water system



oil-water system



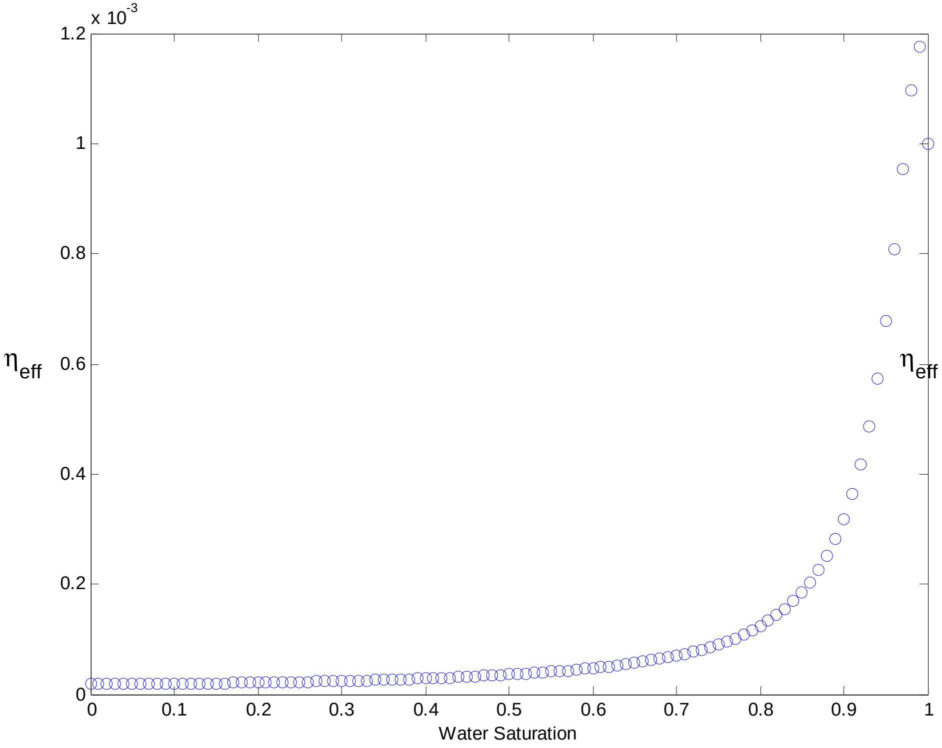
Physical Mechanism

- Effective dynamic shear viscosity parameter for a two-fluid system defined in terms of relative mobilities

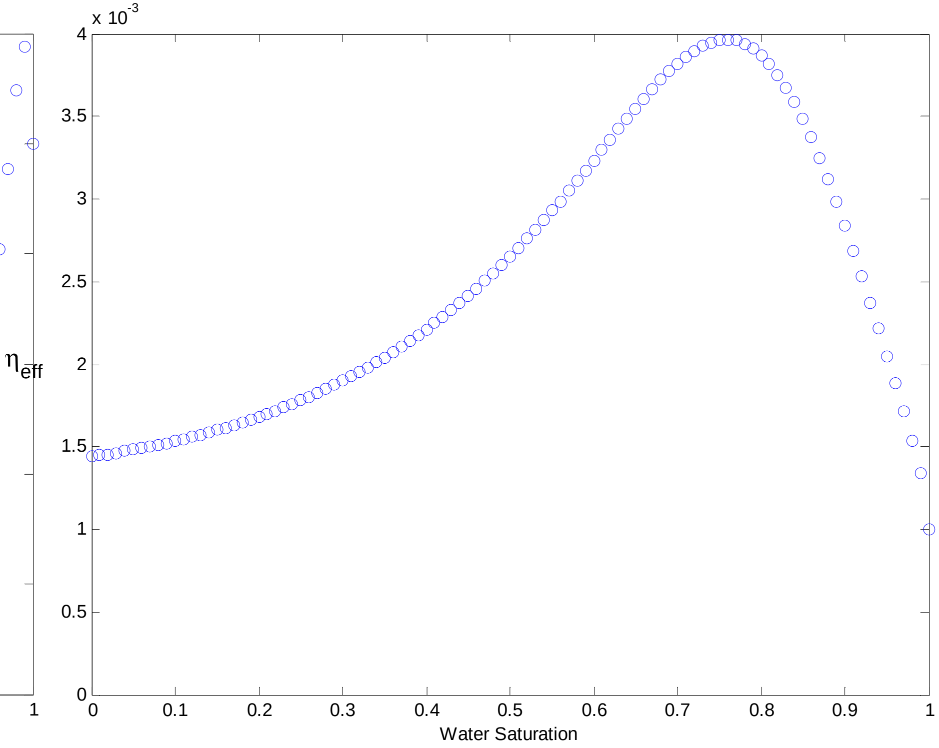
$$\eta_{eff} = \frac{1}{b_1 + b_2} = \frac{\eta_1 \eta_2}{(\eta_2 k_{r1} + \eta_1 k_{r2})}$$

Physical Mechanism

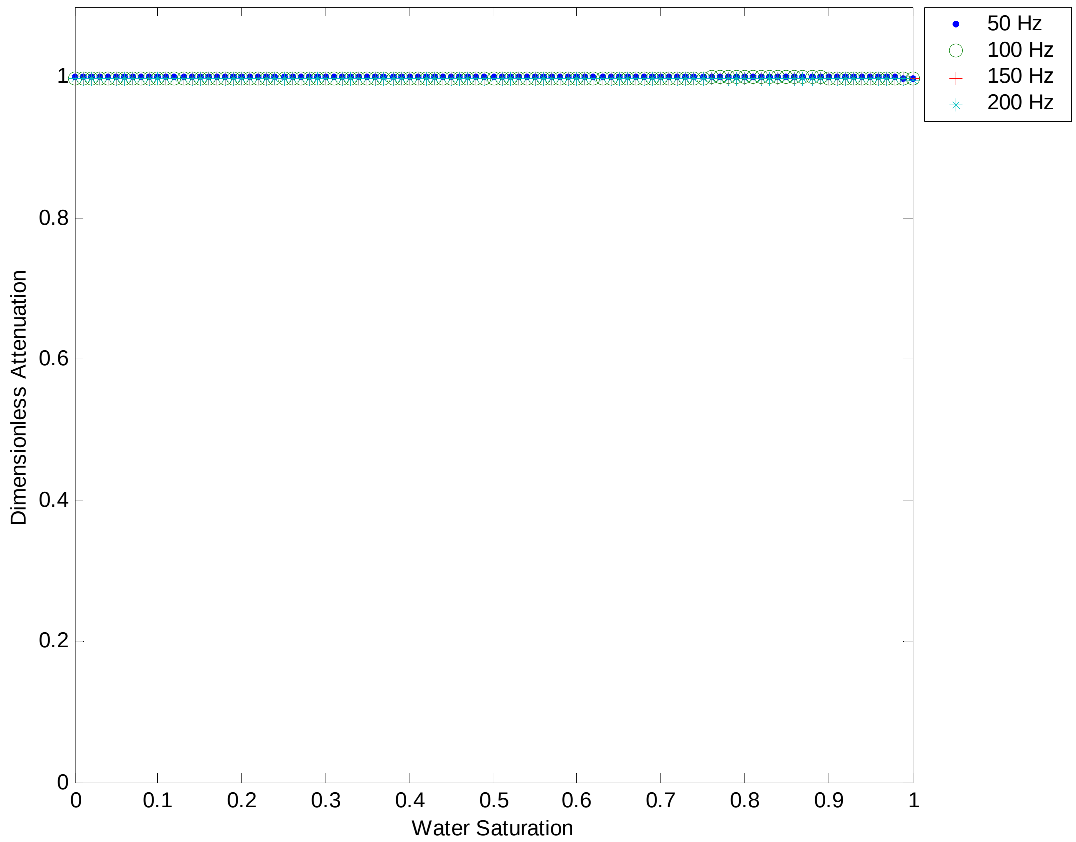
air-water system



oil-water system



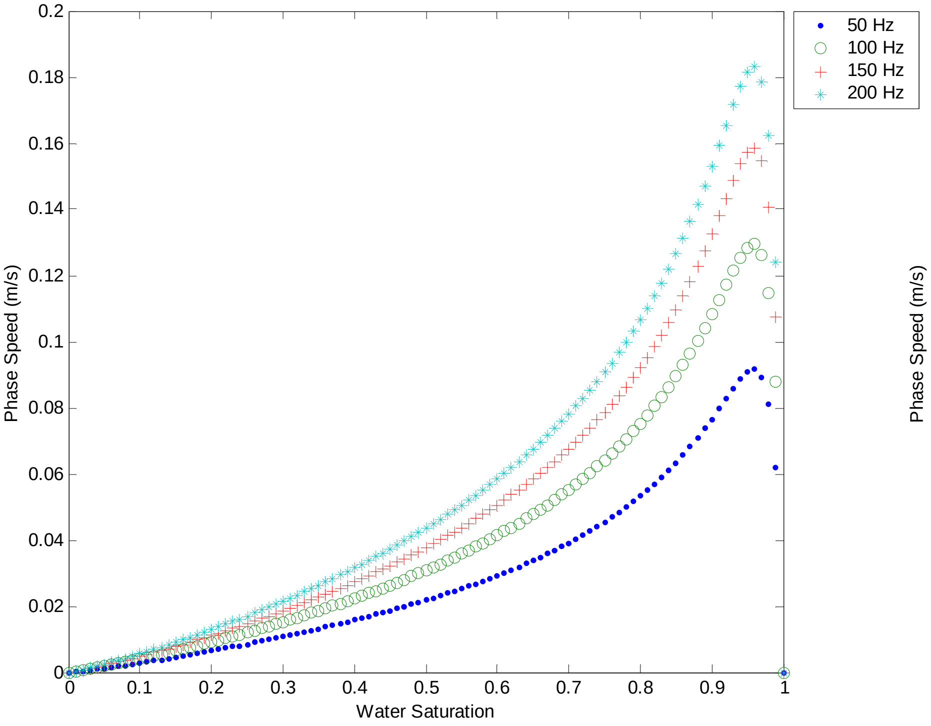
Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Attenuation Coefficient (P2 Wave)



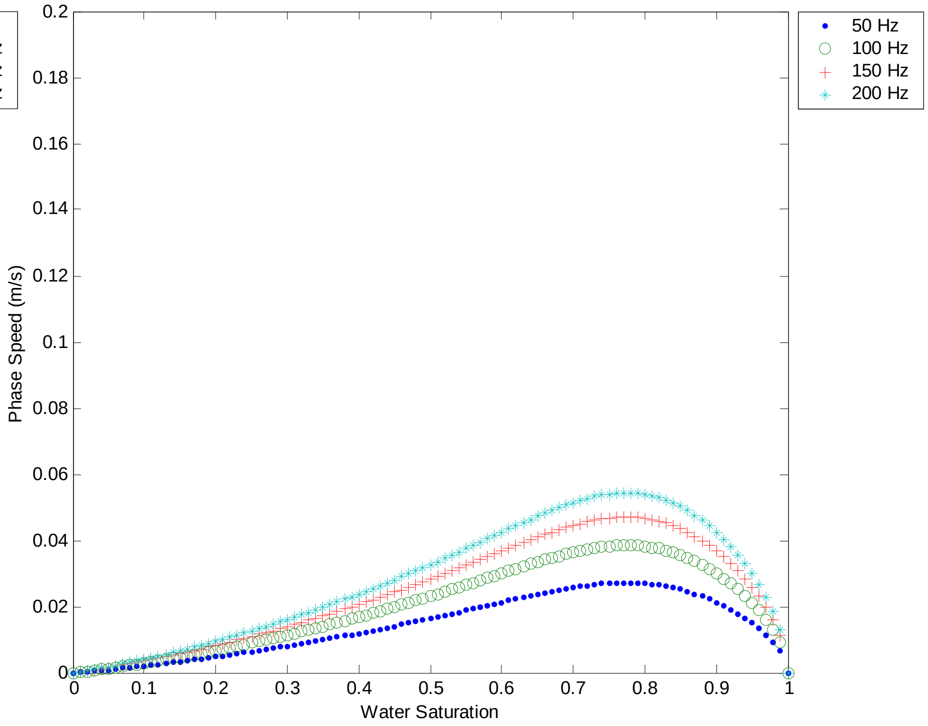
$$\frac{1}{2Q} = \frac{k_i}{k_r}$$

Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Phase Velocity (P3 Wave)

air-water system

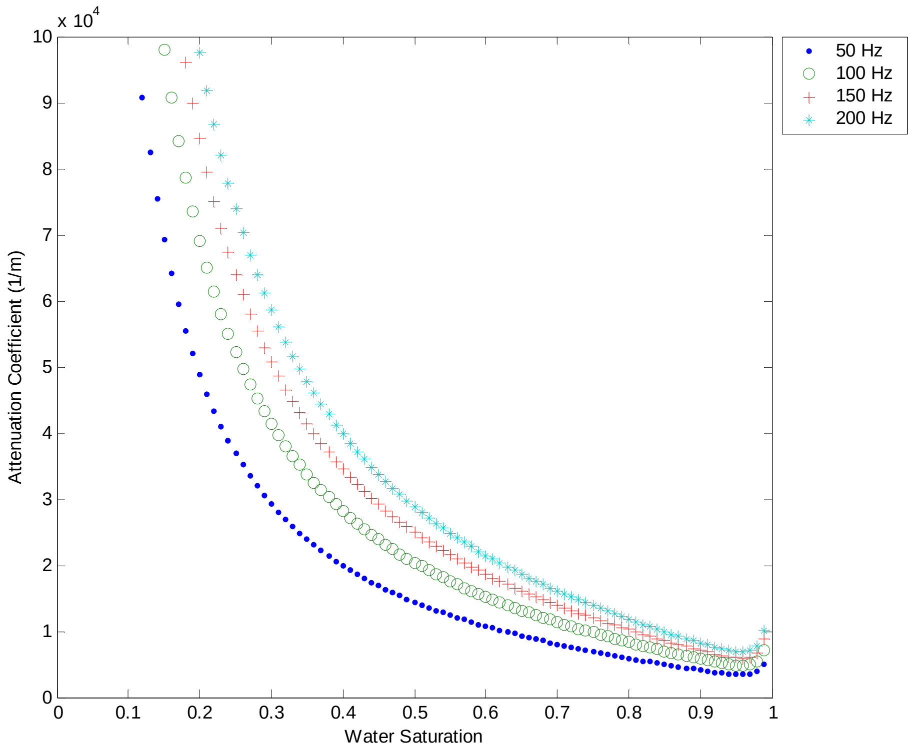


oil-water system

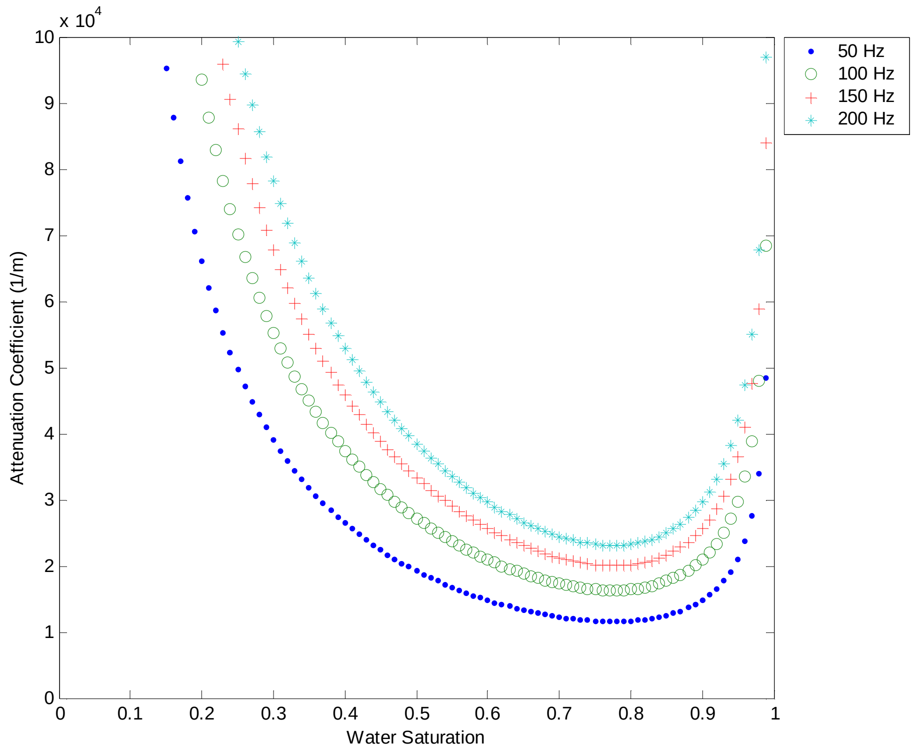


Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Attenuation Coefficient (P3 Wave)

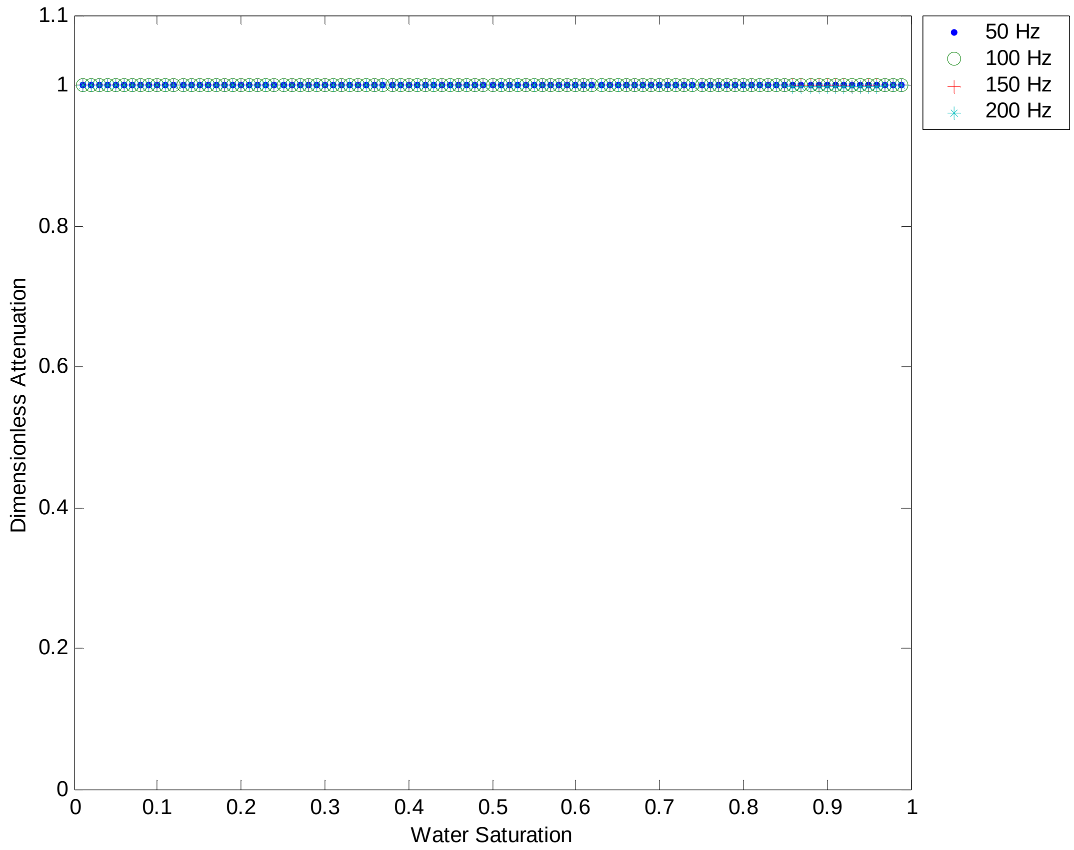
air-water system



oil-water system



Acoustic Wave Propagation in Unconsolidated Fine Sandy Loam Attenuation Coefficient (P3 Wave)



$$\frac{1}{2Q} = \frac{k_i}{k_r}$$

Insights from Numerical Results

- The P1 wave is a sound wave, whereas the P2 and P3 waves are related to dissipative behavior.
- Waves of higher frequency have higher attenuation.
- The P3 wave has the highest attenuation coefficient and the lowest phase velocity.
- The P1 and P2 waves in a two-fluid system are analogous to the fast and slow compressional waves in Biot theory.

Motional Modes

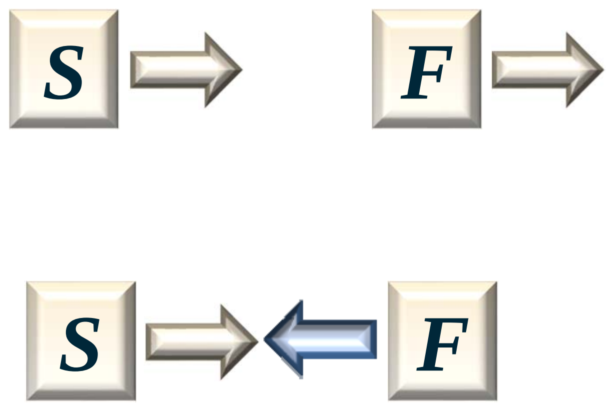
[Lo *et al.*, 2010; *Advances in Water Resources*]

Motional Modes (One-Fluid System)

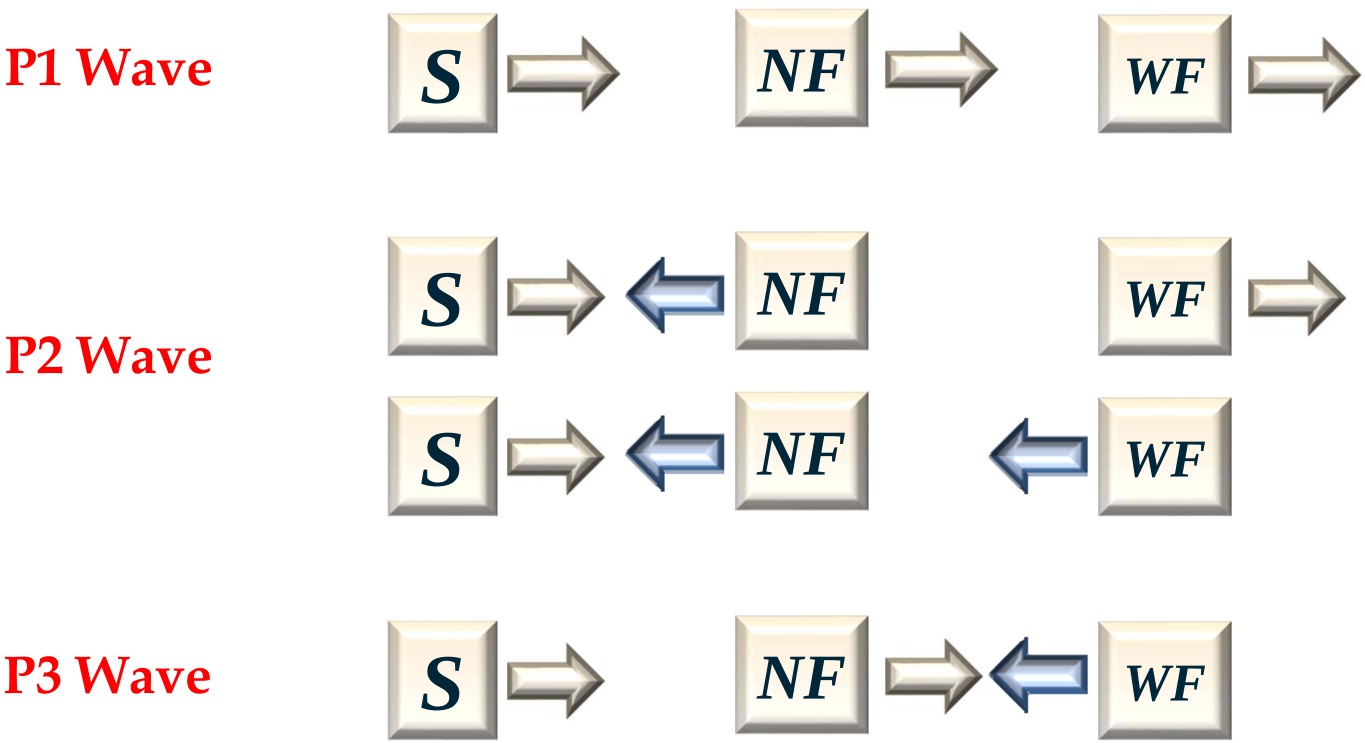
Analysis based on Normal Coordinates !!

P1 Wave

P2 Wave



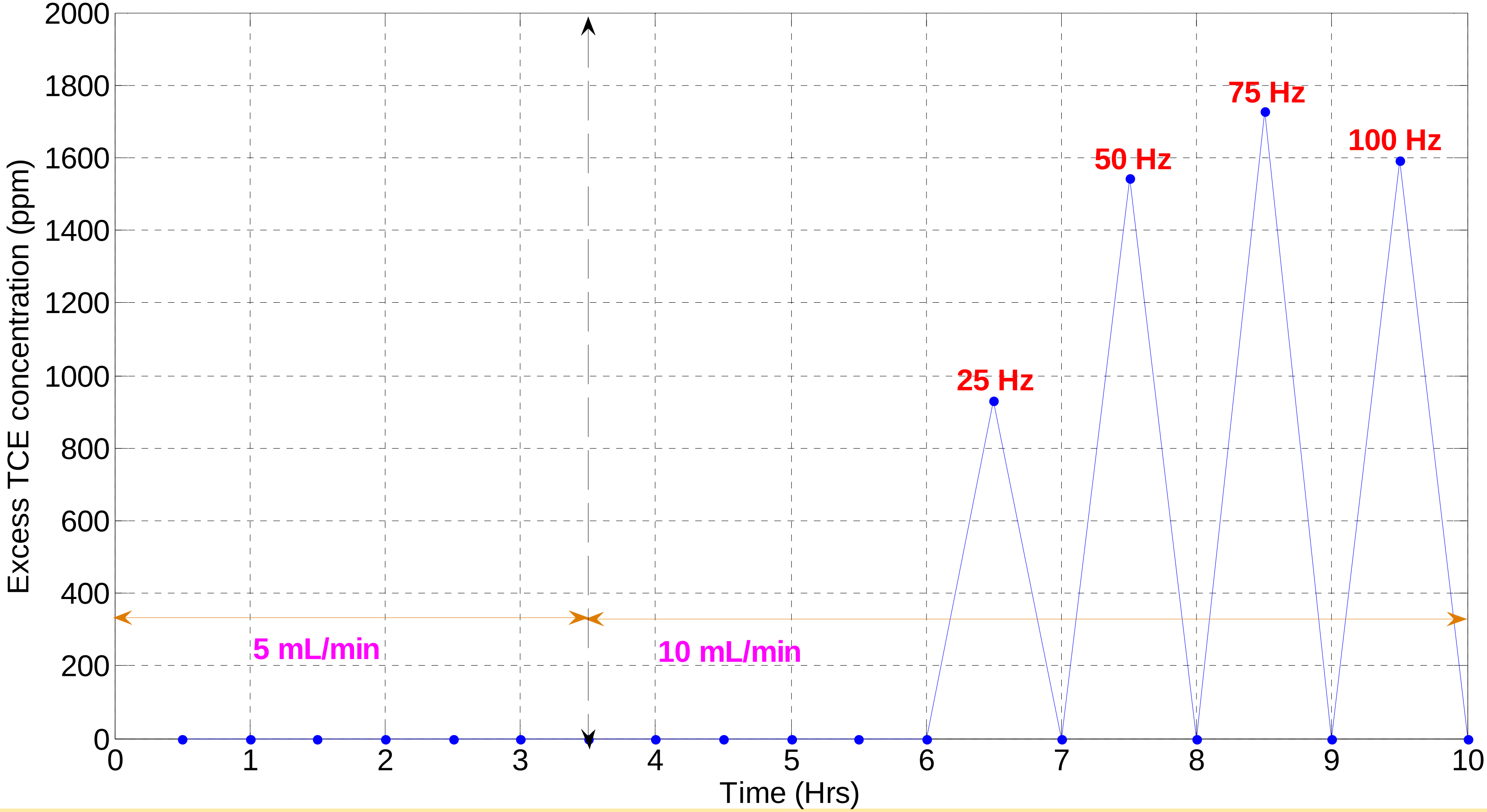
Motional Modes (Two-Fluid System)



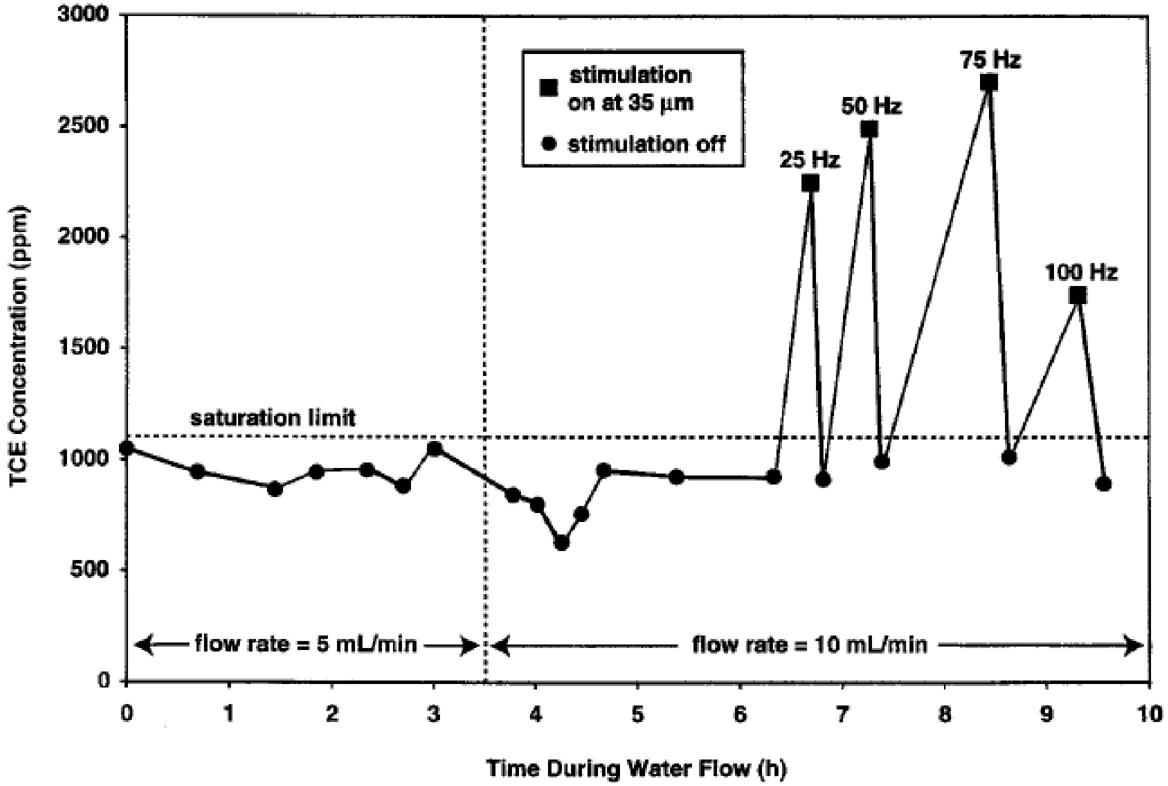
Boundary Value Problem

[Lo *et al.*, 2012, *Journal of Applied Geophysics*]

Boundary Value Problem – Unconsolidated Sand saturated by TCE



Laboratory Observation



Stimulation time: 360s

Pore pressure gradient: 3 kPa/m

Free-phase TCE observed

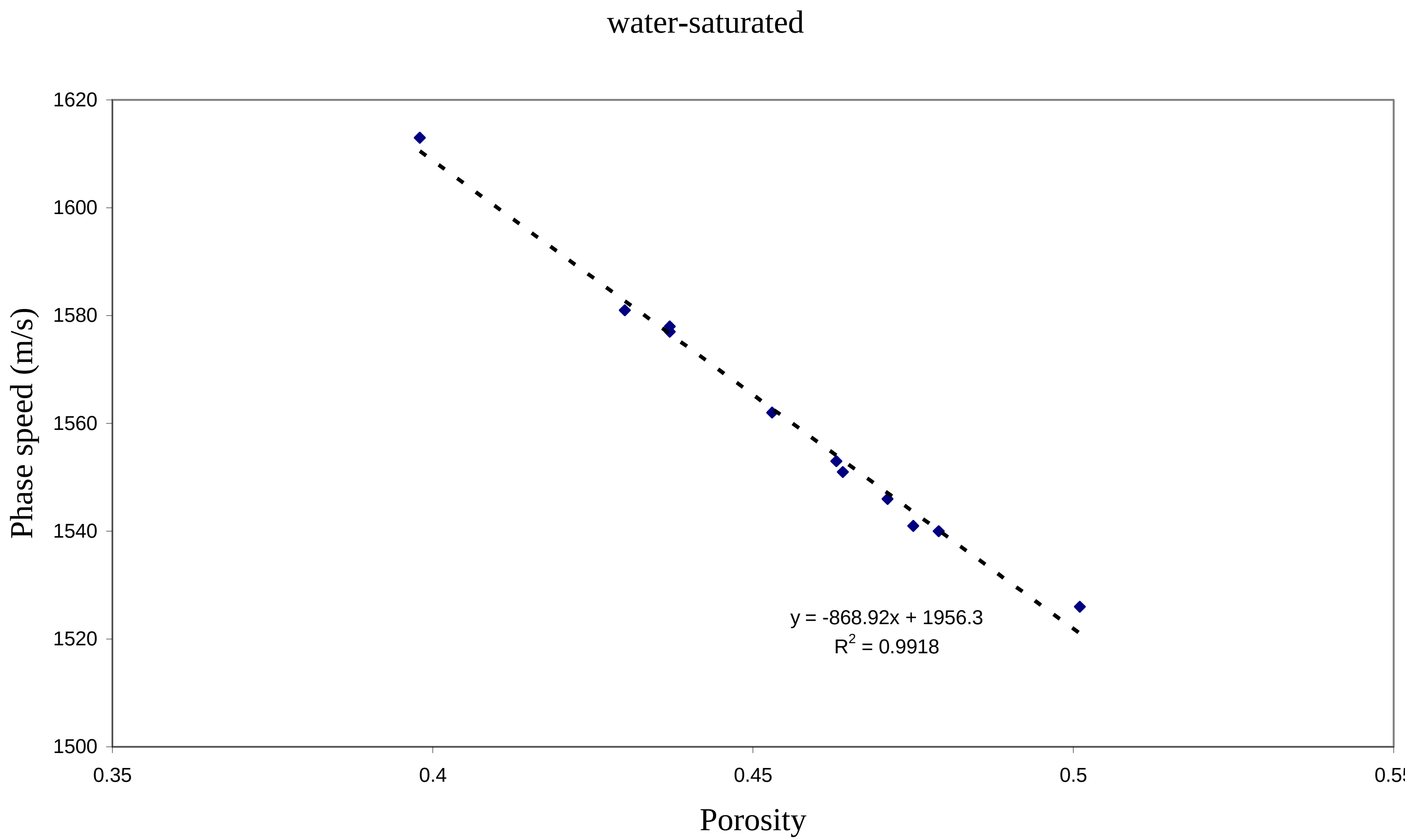
Permeability: 1.1×10^{-10} m^2 (111 d)

Roberts et al., *Environ. Engin. Sci.* **18**(2):67-79 (2001)

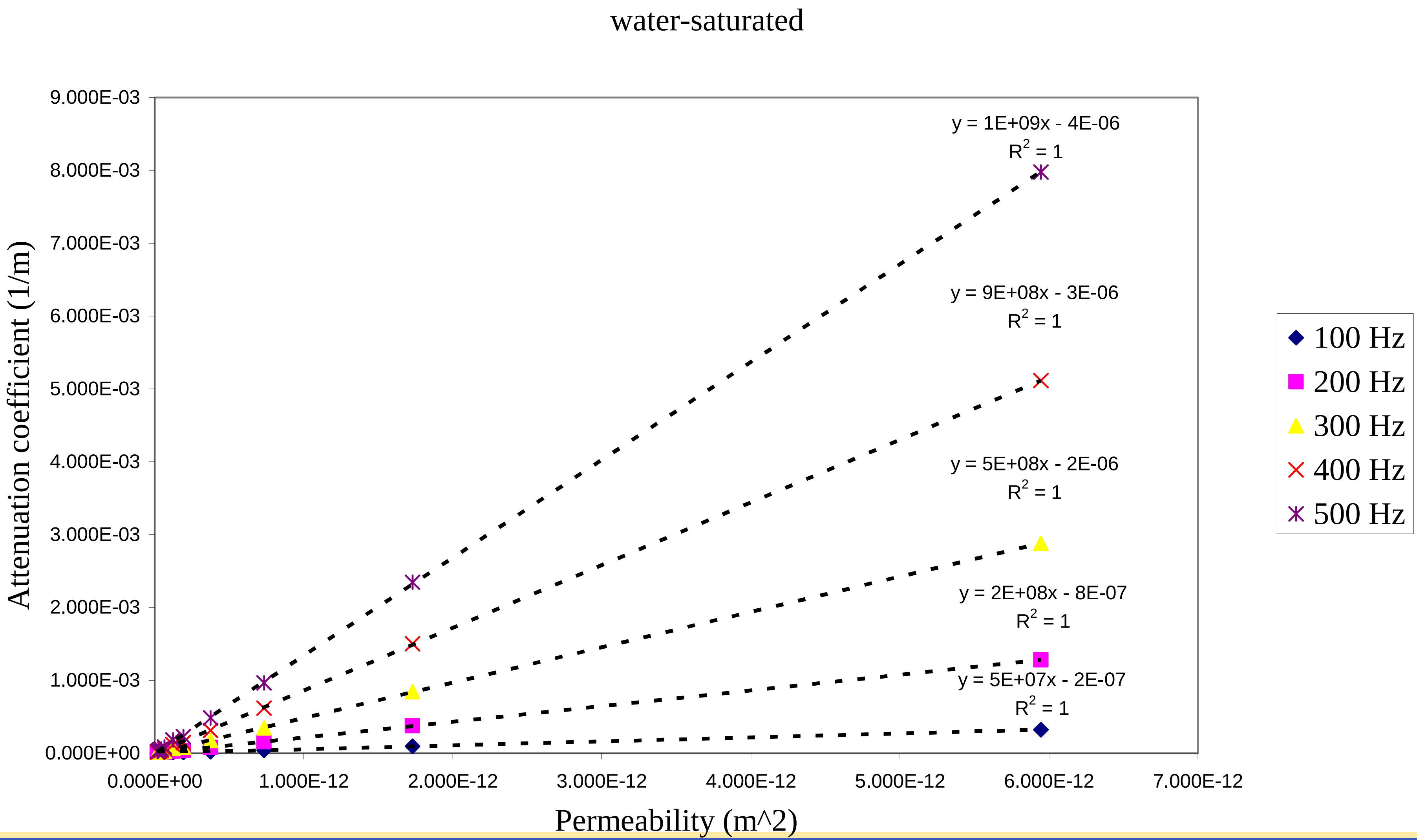
Connecting Acoustic Waves Attributes to Subsurface Hydrological and Geological Parameters

[Lo *et al.*, 2008, 2010; *Journal of Hydrology*]

Quantitative Connection between Porosity and Phase Speed



Quantitative Connection between Permeability and Attenuation Coefficient



Consolidation Theory in Unsaturated Porous Media

[Lo *et al.*, 2014; *Vadose Zone Journal*]

Consolidation

Governing Equations [Lo *et al.*, 2013]:

$$R_{11}\left(\frac{\partial \varepsilon_1}{\partial t}-\frac{\partial e}{\partial t}\right)+R_{22}\left(\frac{\partial \varepsilon_2}{\partial t}-\frac{\partial e}{\partial t}\right)=a_{11}\nabla^2 e+a_{12}\nabla^2\varepsilon_1+a_{13}\nabla^2\varepsilon_2$$

$$-R_{11}\left(\frac{\partial \varepsilon_1}{\partial t}-\frac{\partial e}{\partial t}\right)=a_{21}\nabla^2 e+a_{22}\nabla^2\varepsilon_1+a_{23}\nabla^2\varepsilon_2$$

$$-R_{22}\left(\frac{\partial \varepsilon_2}{\partial t}-\frac{\partial e}{\partial t}\right)=a_{31}\nabla^2 e+a_{32}\nabla^2\varepsilon_1+a_{33}\nabla^2\varepsilon_2$$

Initial Condition

Saturated porous media

- Terzaghi - uncoupled

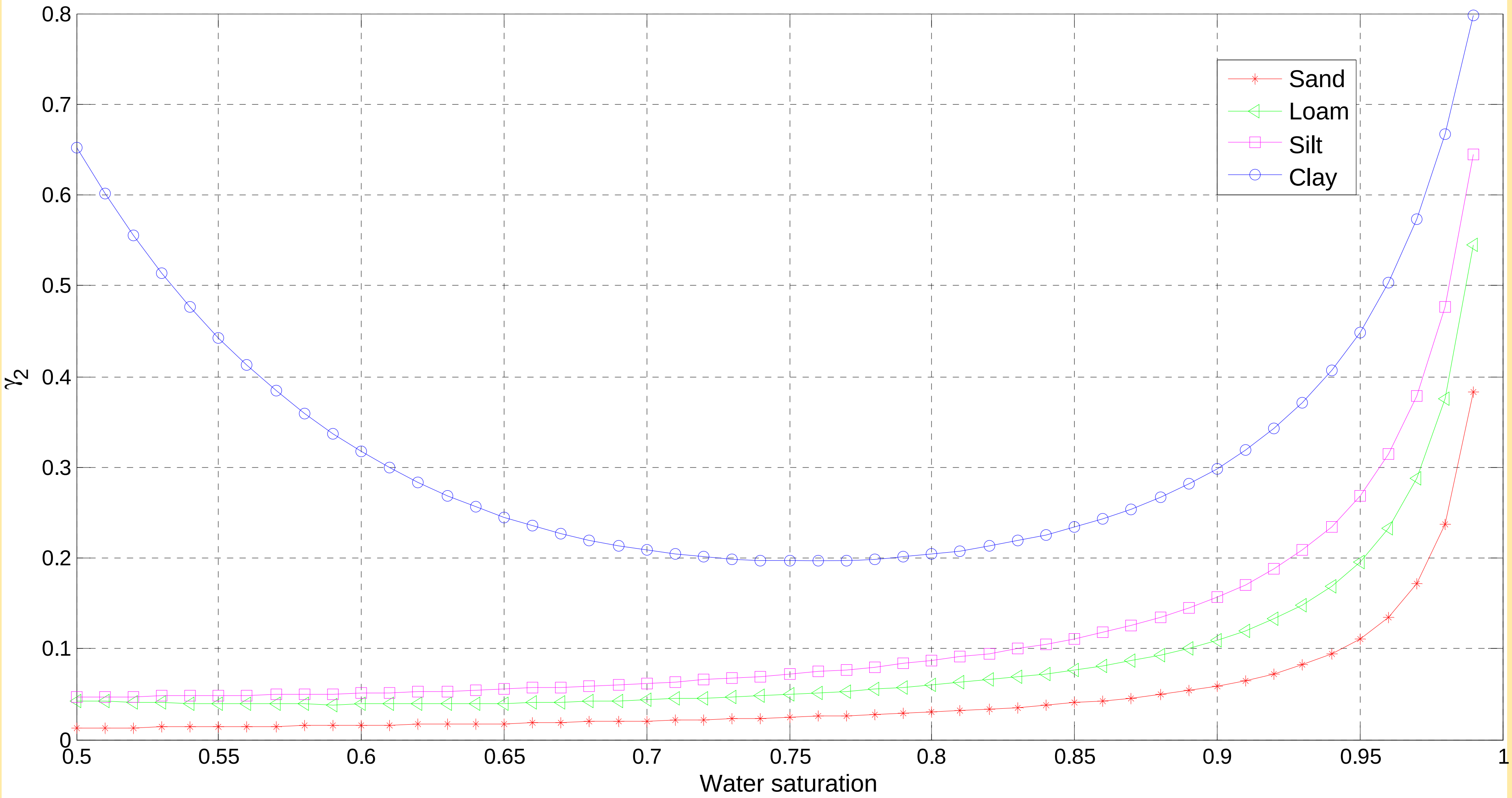
$$p(z,0) = p^*$$

- Biot – coupled

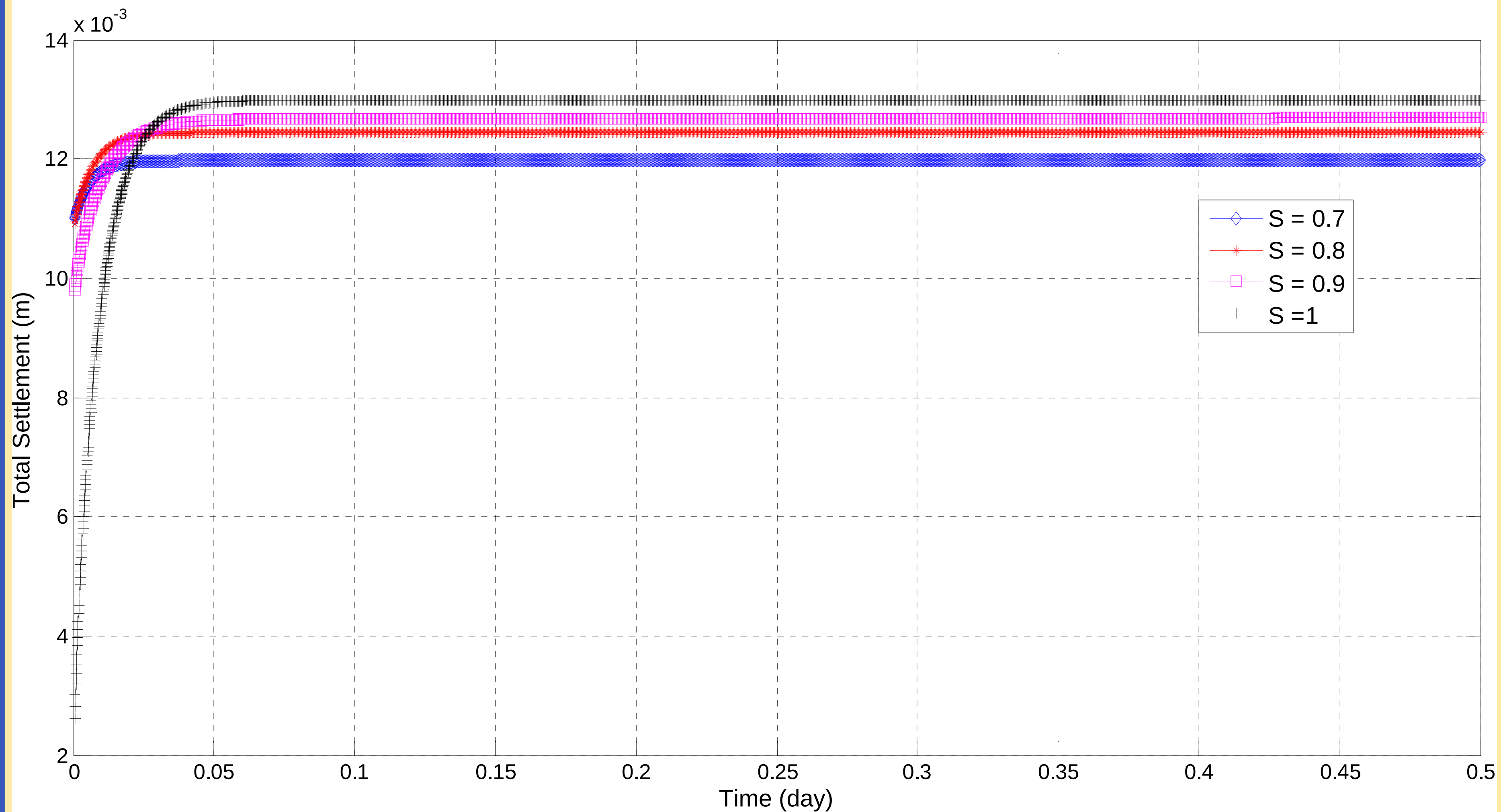
$$p(z,0) = \gamma p^*$$ γ: loading efficiency

Unsaturated porous media

$$p_1(z,0) = \gamma_1 p^* \qquad p_2(z,0) = \gamma_2 p^*$$


Sand
Loam
Silt
Clay
γ_2
Water saturation
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

Total Settlement - Clay



Future Works

- Dynamic Boundary conditions
- Layered Media
- Experimental verification:
  - Laboratory
  - Field