

Modeling and Simulating the Hazardous Flows over Non-Trivial Topography

國立中央大學 水文與海洋科學研究所

7th January 2014

Yih-Chin Tai (戴義欽)

國立成功大學 水利及海洋工程學系

Dept. Hydraulic & Ocean Engineering,

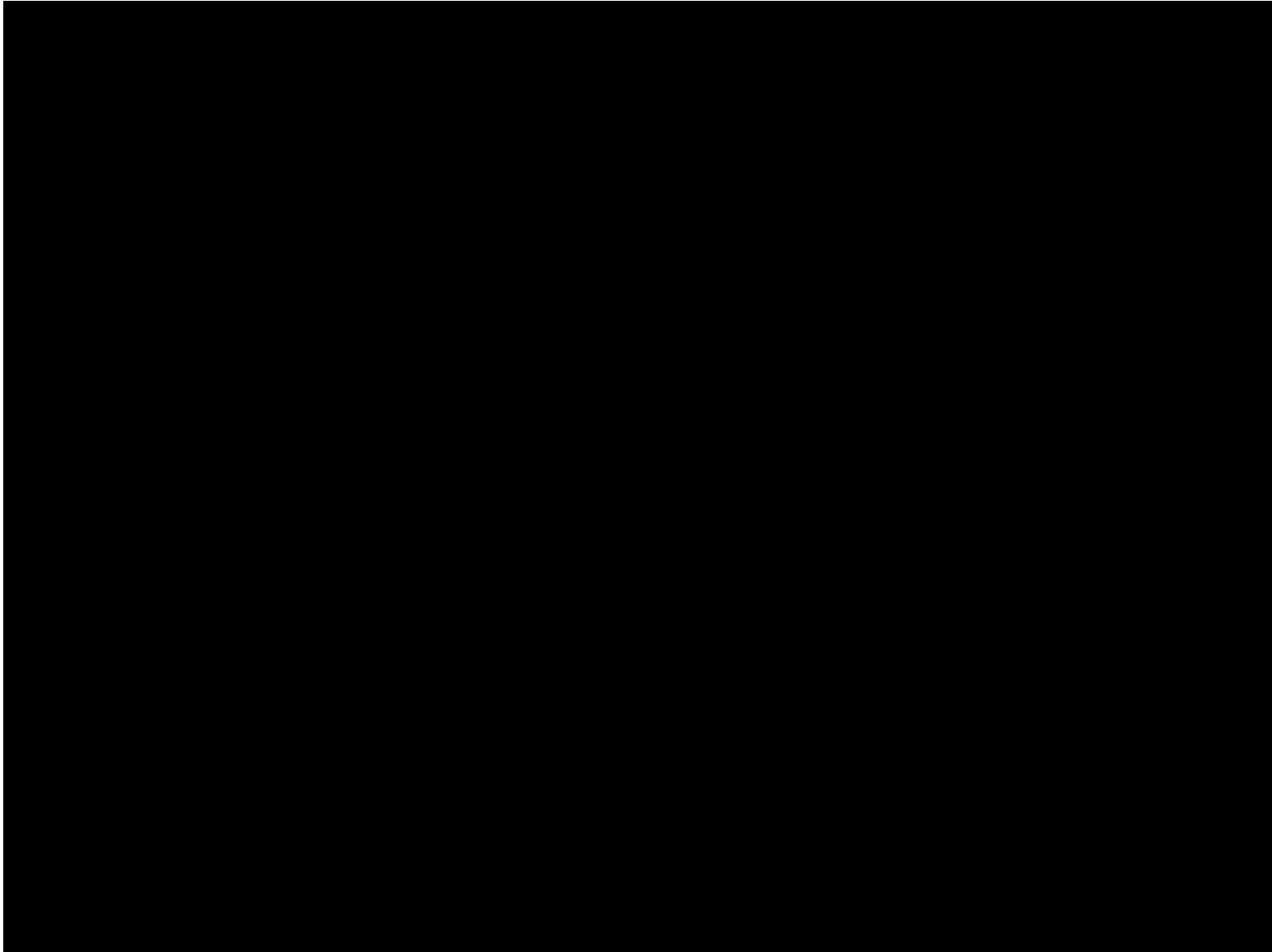
National Cheng Kung University, Tainan, Taiwan.

Contents

- Motivation and background
- Coordinate system for general topography
- Model equations and Numerical example
- Reconstruction of the Hsiaolin landslide catastrophe
- Concluding remarks

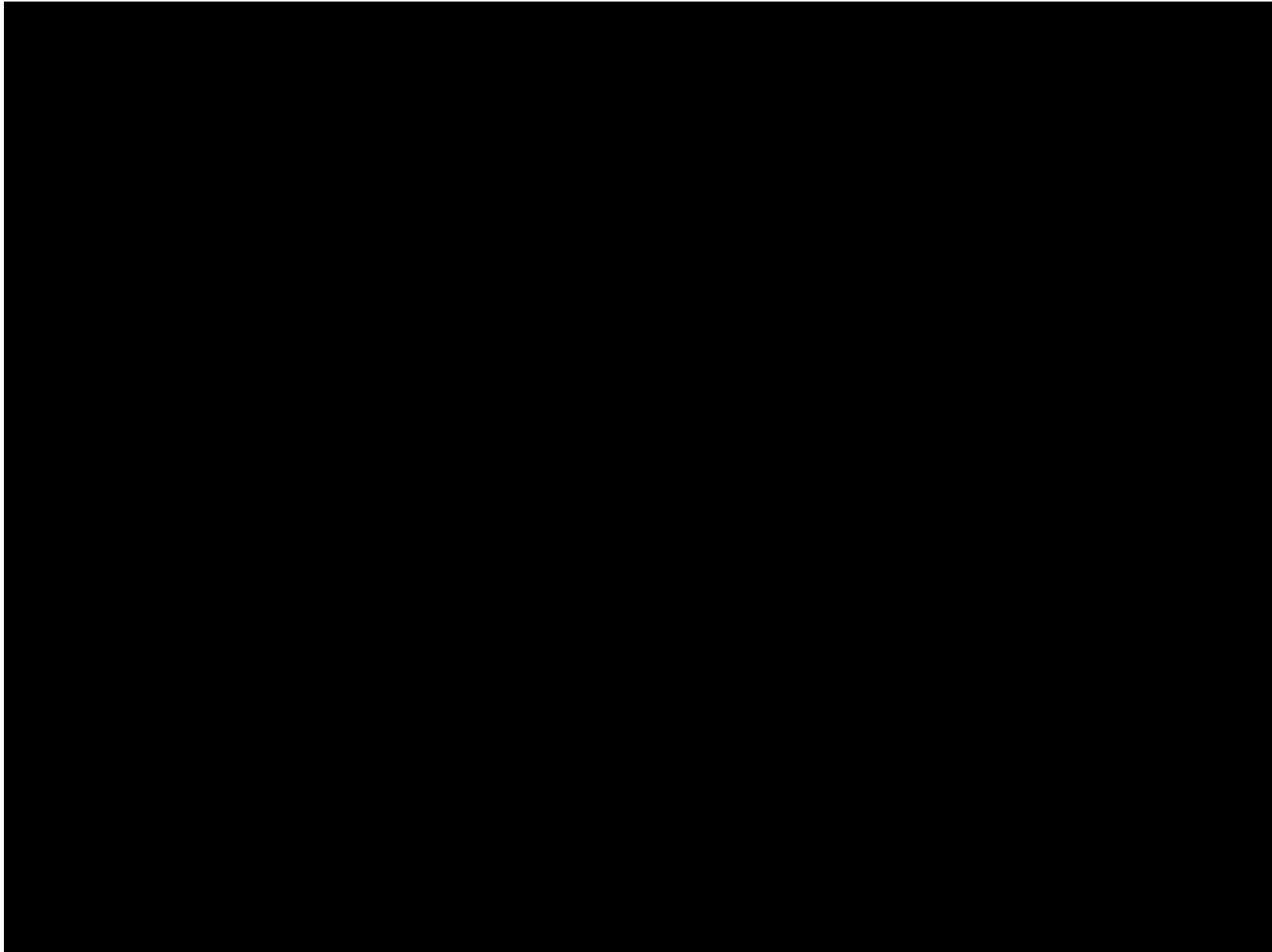
Earthquake induced landslide in middle Taiwan, 2nd June 2013

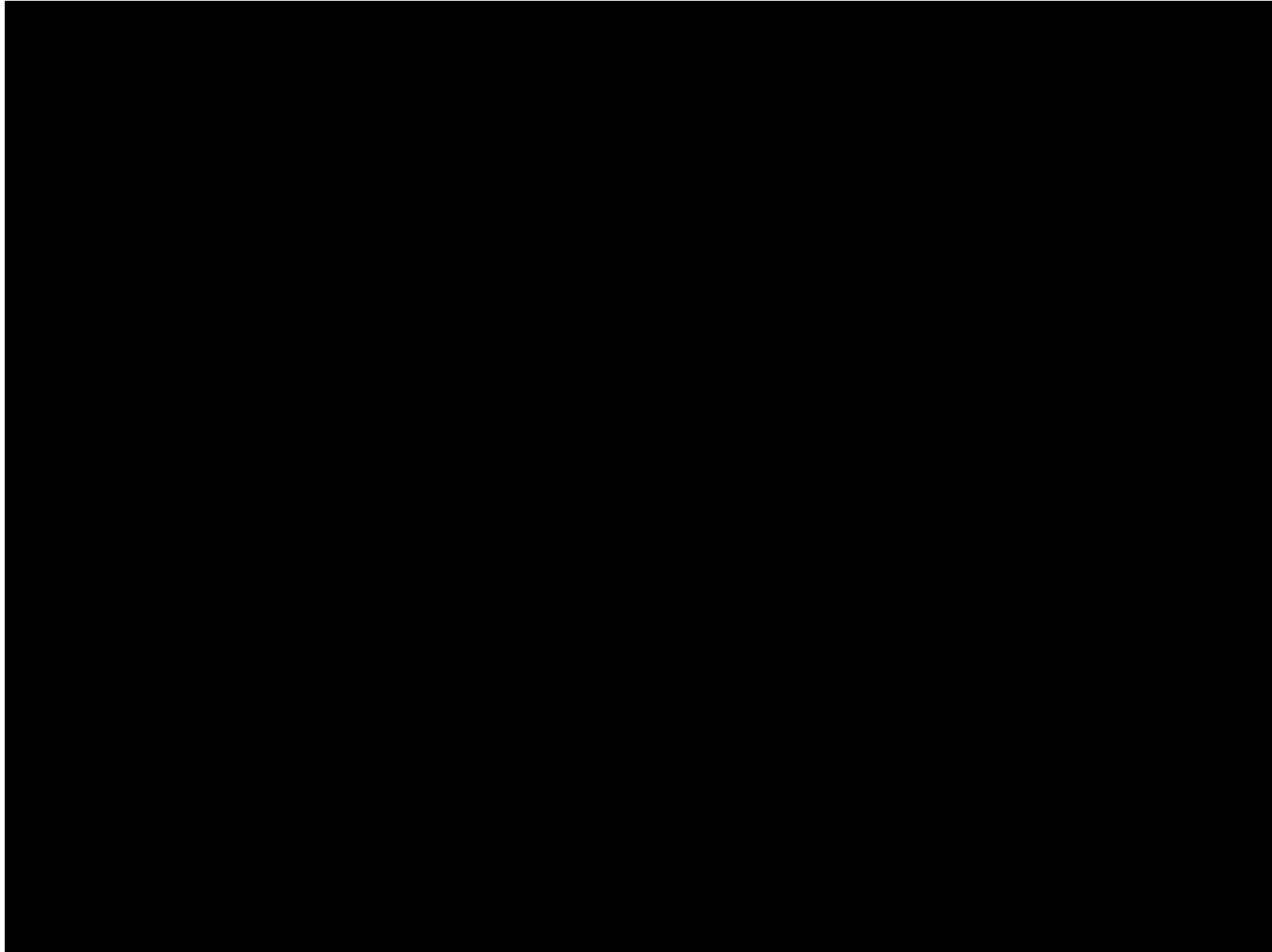
(Magnitude 6.3, Depth 10 km) ([from Youtube](#))



Heavy rainfall induced landslide in southern Taiwan, 1st September 2013

(after the Typhoon Kong-Rey 康芮) ([from Youtube](#))





Hsiaolin catastrophe (2009)

- Typhoon Morakot
- Aug. 7th-10th 2009
- > 2,000 mm accumulated precipitation in southern Taiwan
- Total 724 deaths (474 in Hsiaolin catastrophe)



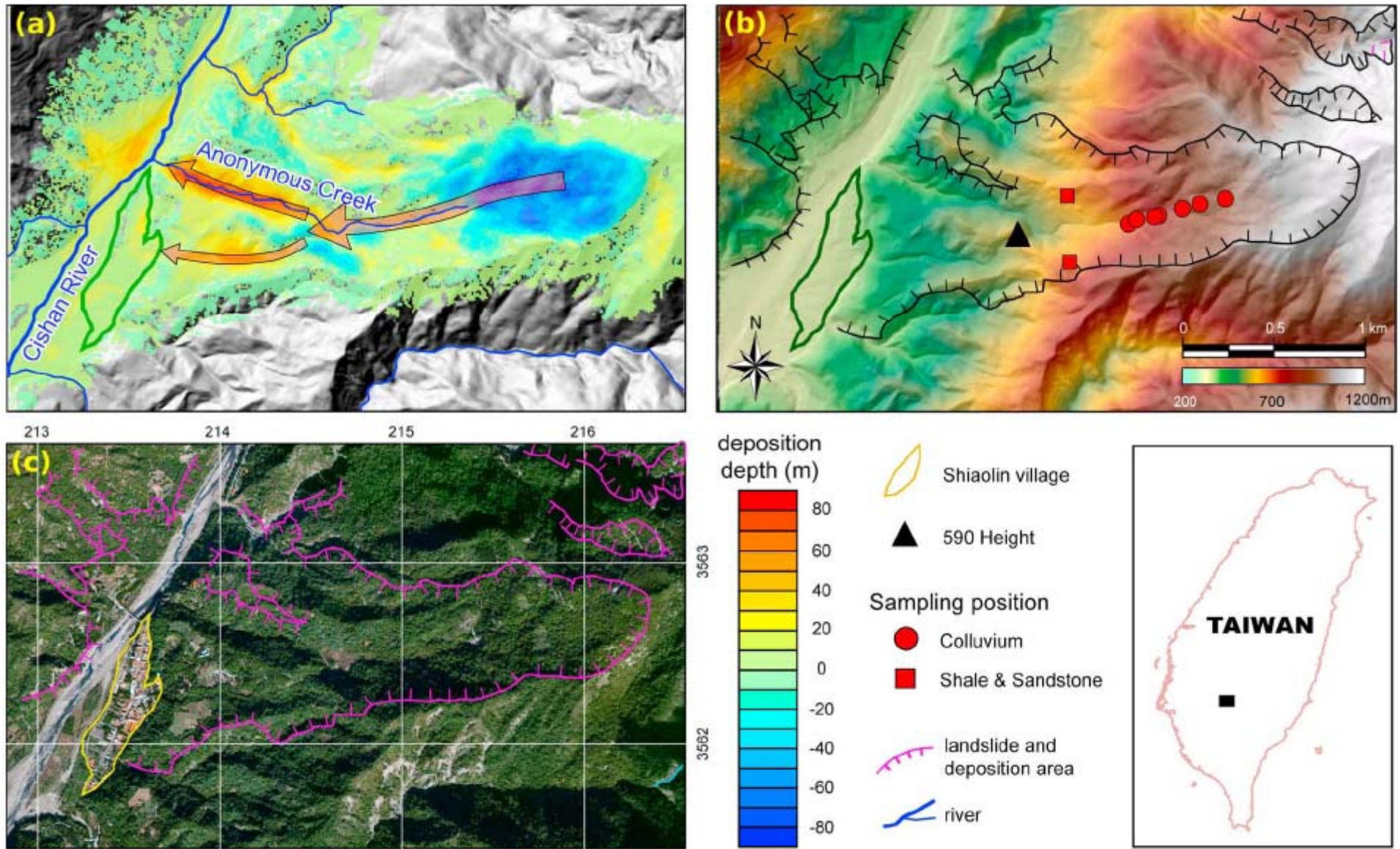
before

[八八水災圖片情報網](#)

after



Volume: ca. $24 \pm 2 \text{ Mm}^3$
Run-out distance: c.a. 2,700 m
Descending elevation: c.a. 800 m

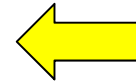


Volume: ca. $24 \pm 2 \text{ Mm}^3$, Run-out distance: c.a. 2,700 m
 Descending elevation: c.a. 800 m

Three main challenges in simulating the behavior:

- Gravity-driven flows over complex topography

- *an appropriate coordinate system (local curvature)*
- *an appropriate mesh system for the complex topography (depth-integration or not)*

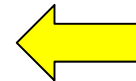


- Characteristics of the flow

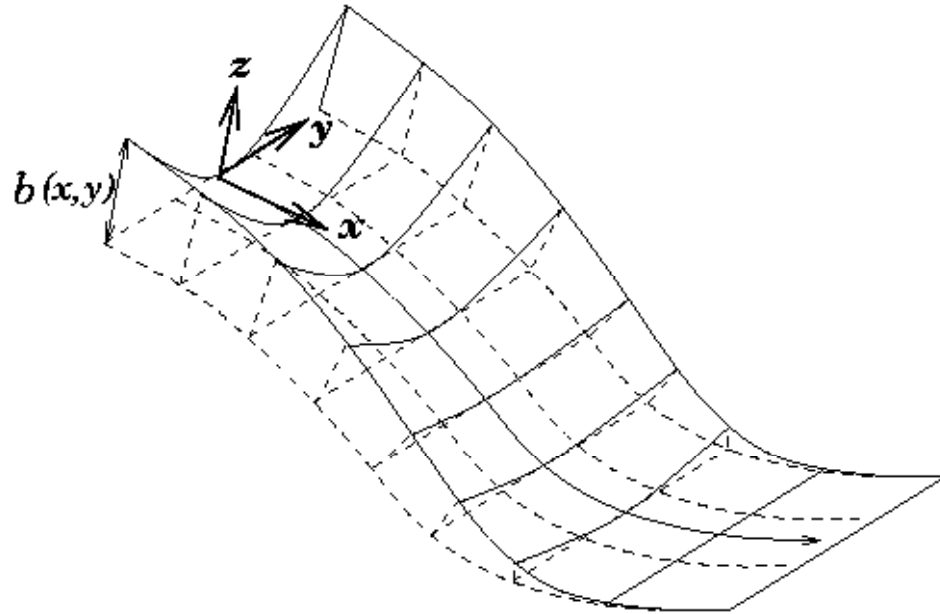
- *constitutive relation (rheology)*
- *single-phase or mixture theory*

- Deformable basal surface

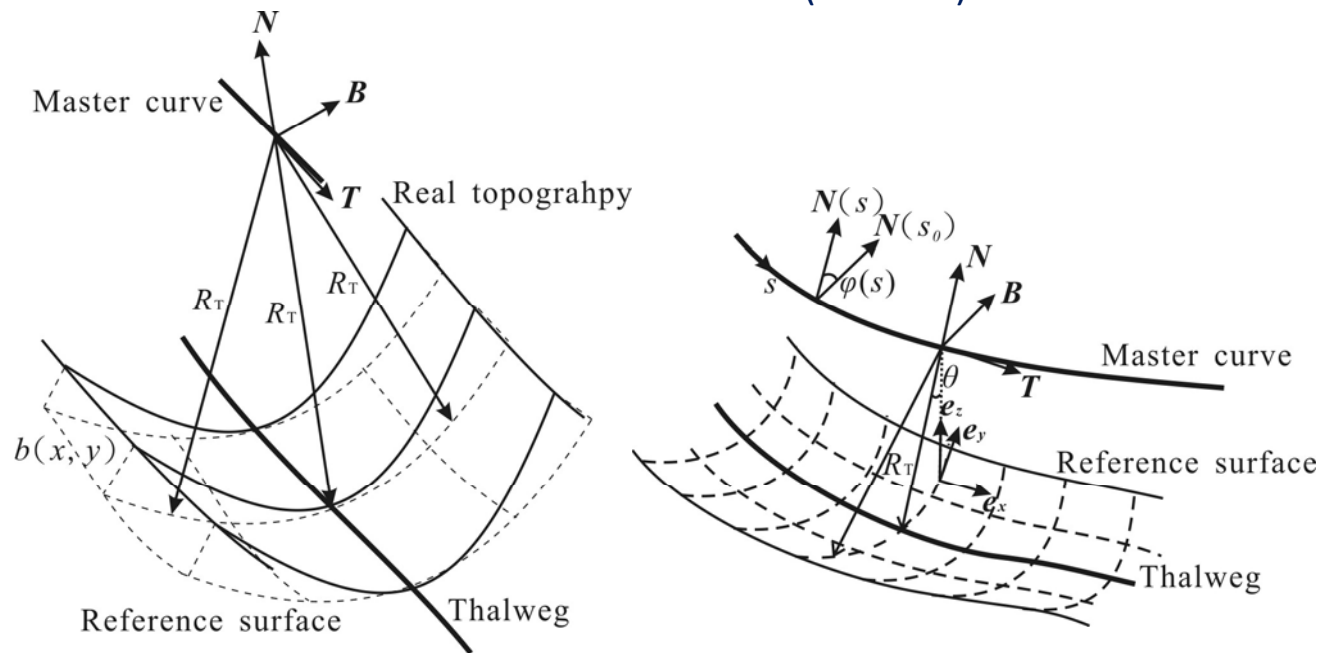
- *entrainment and deposition*
- *non-material surface*
- *evolution of the basal surface*

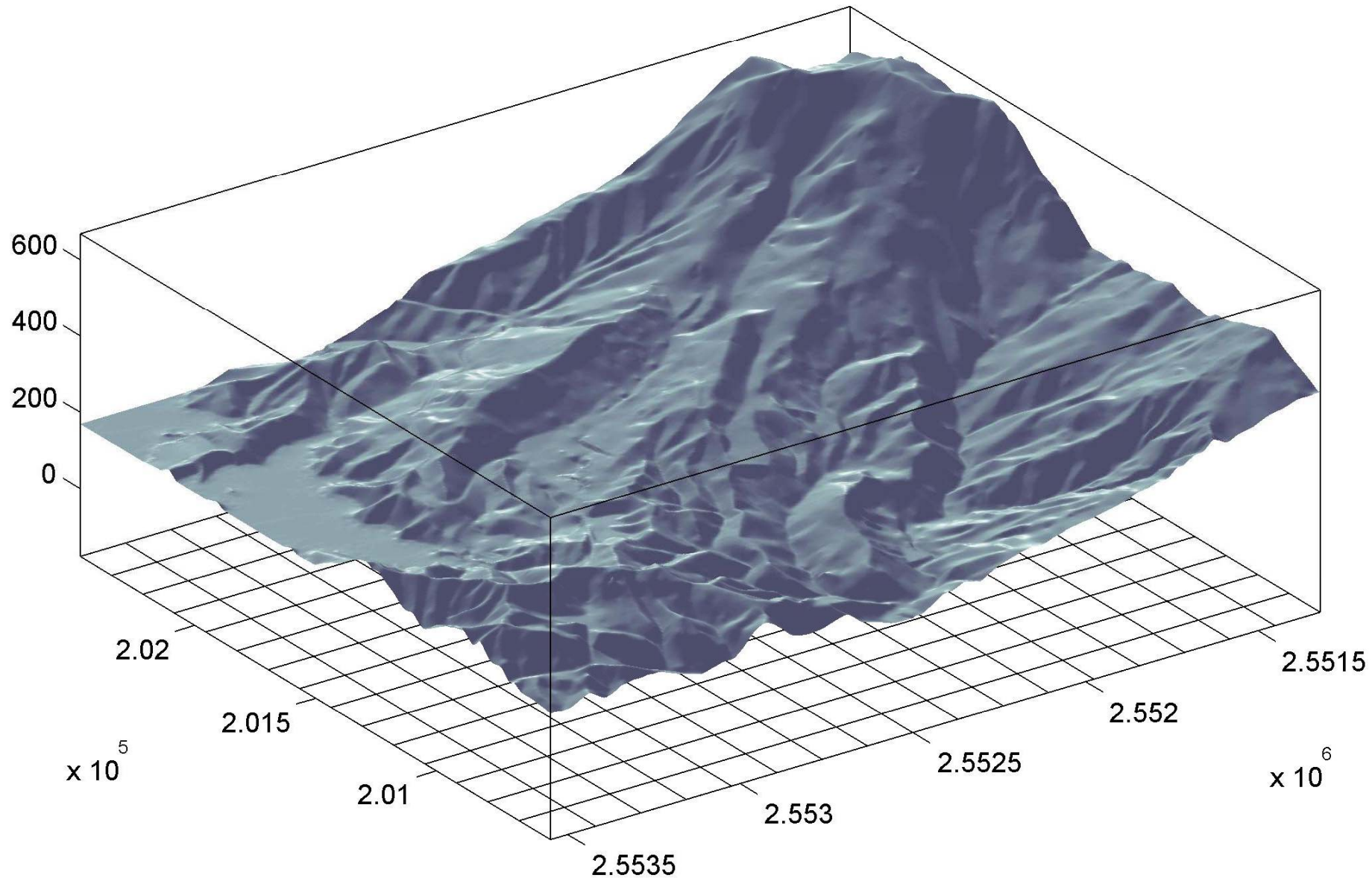


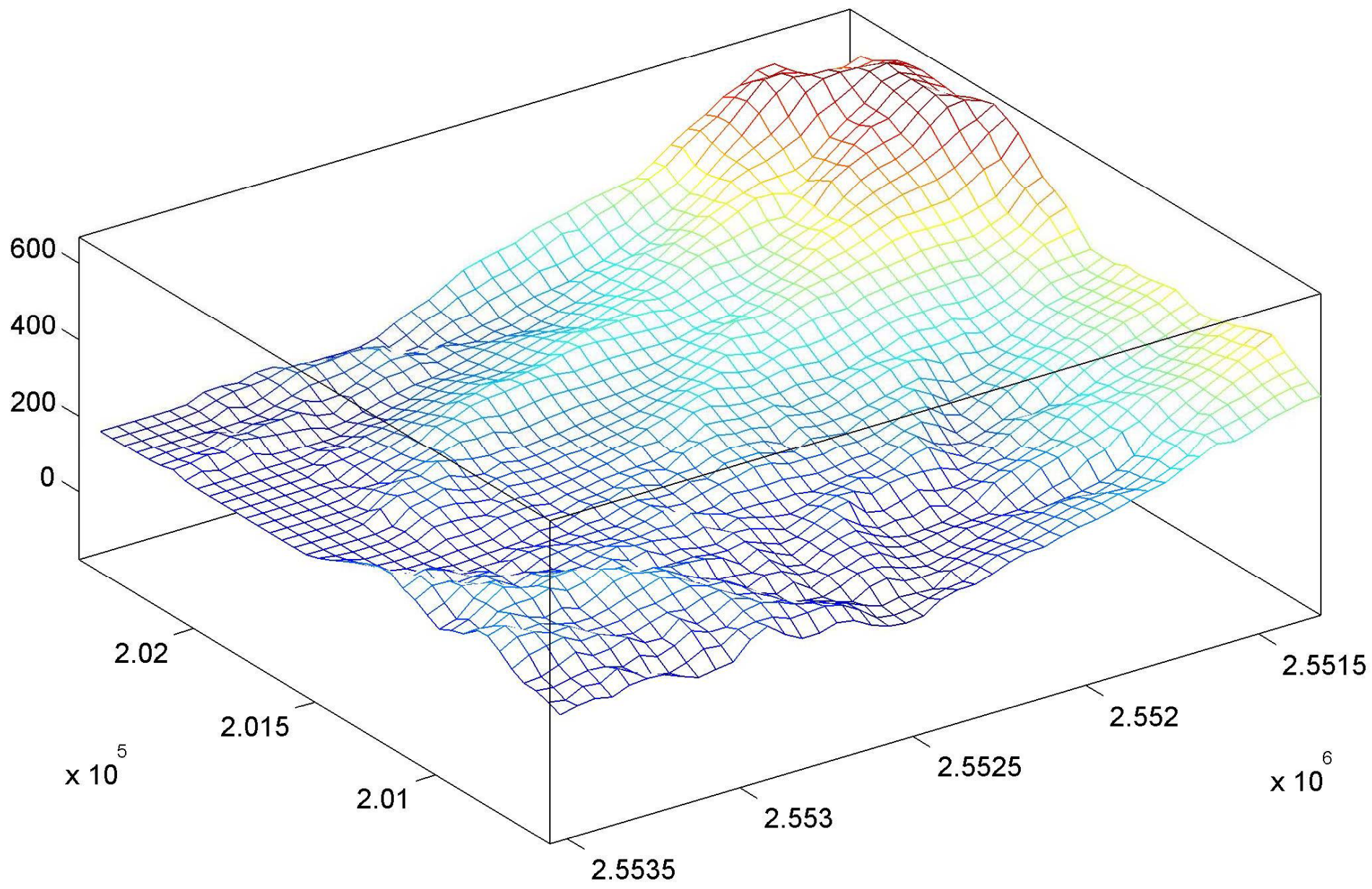
- **Coordinates of SH-Type** (1989 – 1999)



- **Curved and Twisted Coordinates** (2003 –)







Coordinates for general topography (BW 2004)

- **Topography-fitted coordinates,**
where the flow thickness is much smaller than the curvature radius

- **Basal topography**

$$z - b(x, y, t) = 0$$

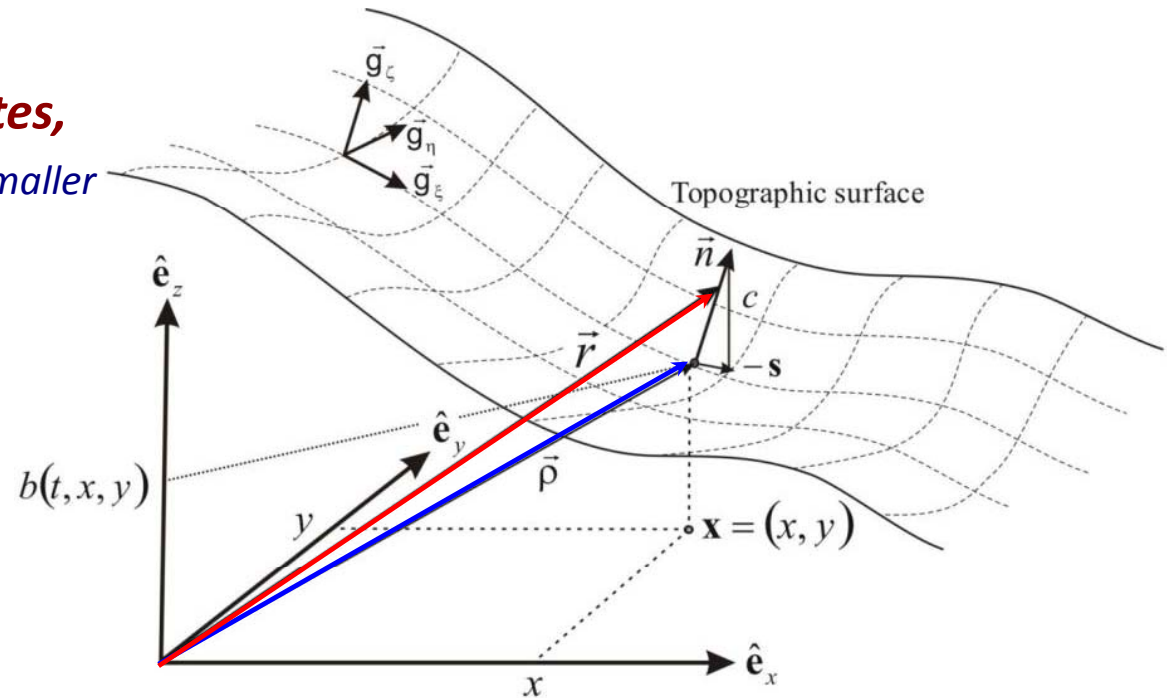
- **Basal normal vector**

$$\vec{n} = n_x \mathbf{e}_x + n_y \mathbf{e}_y + n_z \mathbf{e}_z$$

$$\text{where } n_x = -c \partial_x b, \quad n_y = -c \partial_y b, \quad n_z = c = \left(1 + (\partial_x b)^2 + (\partial_y b)^2\right)^{-1/2}$$

- **Position vector in the flow**

$$\begin{aligned} \vec{r} &\equiv r_x \mathbf{e}_x + r_y \mathbf{e}_y + r_z \mathbf{e}_z = \vec{\rho} + \zeta \vec{n} \\ &= (x + \zeta n_x) \mathbf{e}_x + (y + \zeta n_y) \mathbf{e}_y + (b + \zeta n_z) \mathbf{e}_z \end{aligned}$$



Coordinate Transforms

- **Coordinate Transformation** (e.g. Hui 2004, Hui 2007)

$$d\vec{r} = \partial_{\lambda}\vec{r} d\lambda + \partial_{\xi}\vec{r} d\xi + \partial_{\eta}\vec{r} d\eta + \partial_{\zeta}\vec{r} d\zeta = \vec{Q} d\lambda + \tilde{\Omega} d\vec{\xi} \quad dt = d\lambda$$

where

\vec{Q} is the local velocity of the coordinate

$\tilde{\Omega} = \nabla_{\vec{\xi}}\vec{r}$ is the Jacobian matrix

- **The choice of the coordinate velocity**

\vec{Q} is zero \Rightarrow Eulerian description

\vec{Q} is chosen to the velocity of the fluid particle \Rightarrow Lagrangian description

\vec{Q} is set to the basal entrainment/deposition rate
 \Rightarrow **Terrain-following coordinate**

Equations of Conservative Laws

- *Equations of mass and momentum conservation*

$$\begin{aligned} \partial_t u^i &= 0 \\ \partial_t u^i + \partial_j (u^i u^j - t^{ij}) &= g^i \end{aligned} \quad \text{for } i, j \in \{x, y, z\}$$

$$\begin{aligned} \vec{v} &= u^x \mathbf{e}_x + u^y \mathbf{e}_y + u^z \mathbf{e}_z \\ &= u^\xi \mathbf{g}_\xi + u^\eta \mathbf{g}_\eta + u^\zeta \mathbf{g}_\zeta \end{aligned}$$

- *Transformed equations in the topography-fitted coordinates*

$$\begin{aligned} \partial_\lambda J + \partial_m u^m &= 0 \\ \partial_\lambda (J u^i) + \partial_m (J u^i (u^m - Q^m) - J \Sigma^{im}) &= J g^i \end{aligned}$$

$$J = \det \tilde{\Omega}$$

$$dt = d\lambda$$

$$\text{for } i, j \in \{x, y, z\}, \quad m, n \in \{\xi, \eta, \zeta\}$$

- *The stress term*

$$\Sigma^{in} = \Omega_m^i t^{mn}$$

Remark: A similar treatment for Euler equations can be found in *Vinolur (1974) J. Comput. Phys.*

Approximation and Depth Integration

- *Jacobian matrix*

$$\tilde{\mathbf{\Omega}} = \begin{pmatrix} (\mathbf{I} - \zeta \partial_{\mathbf{x}} \mathbf{s}) \partial_{\xi} \mathbf{x} & -\mathbf{s} \\ (\partial_{\xi} b)^T + (\zeta \partial_{\xi} c)^T & c \end{pmatrix} = \begin{pmatrix} \partial_{\xi} \mathbf{x} & -\mathbf{s} \\ (\partial_{\xi} b)^T & c \end{pmatrix} + O(\varepsilon^{1+\alpha}) = \tilde{\mathbf{\Omega}}_b + O(\varepsilon^{1+\alpha})$$

$$J = \det \tilde{\mathbf{\Omega}} = J_b + O(\varepsilon^{1+\alpha})$$

where

$$\mathbf{s} = \begin{pmatrix} s_x \\ s_y \end{pmatrix} = \begin{pmatrix} c \partial_x b \\ c \partial_y b \end{pmatrix}, \quad \partial_{\xi} b = \begin{pmatrix} \partial_{\xi} b \\ \partial_{\eta} b \end{pmatrix}, \quad \partial_{\xi} c = \begin{pmatrix} \partial_{\xi} c \\ \partial_{\eta} c \end{pmatrix},$$
$$\partial_{\mathbf{x}} \mathbf{s} = \begin{pmatrix} \partial_x s_x & \partial_y s_x \\ \partial_x s_y & \partial_y s_y \end{pmatrix}, \quad \partial_{\xi} \mathbf{x} = \begin{pmatrix} \partial_{\xi} x & \partial_{\eta} x \\ \partial_{\xi} y & \partial_{\eta} y \end{pmatrix}.$$

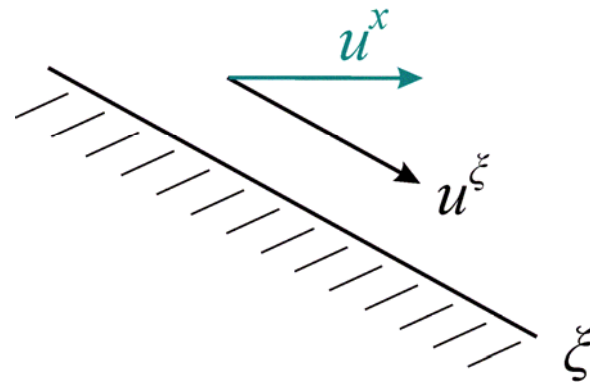
Depth-integrated equations

$$\frac{\partial}{\partial \lambda} (J_b h) + \frac{\partial}{\partial \xi} (J_b h \bar{u}^\xi) + \frac{\partial}{\partial \eta} (J_b h \bar{u}^\eta) = -J_b E$$

$$\begin{aligned} \frac{\partial}{\partial \lambda} \begin{pmatrix} J_b h \bar{u}^x \\ J_b h \bar{u}^y \end{pmatrix} + \frac{\partial}{\partial \xi} \begin{pmatrix} J_b \varpi_{\parallel} h \bar{u}^x \bar{u}^\xi + J_b h \bar{p} M_{11} \\ J_b \varpi_{\parallel} h \bar{u}^y \bar{u}^\xi + J_b h \bar{p} M_{21} \end{pmatrix} \\ + \frac{\partial}{\partial \eta} \begin{pmatrix} J_b \varpi_{\parallel} h \bar{u}^x \bar{u}^\eta + J_b h \bar{p} M_{12} \\ J_b \varpi_{\parallel} h \bar{u}^y \bar{u}^\eta + J_b h \bar{p} M_{22} \end{pmatrix} = -J_b E \begin{pmatrix} u_b^x \\ u_b^y \end{pmatrix} + \mathbf{s}^\xi \end{aligned}$$

- **Material equation**
- **Boundary conditions**
 - Drag free at flow surface.
 - Coulomb friction at basal surface (or Voellmy's law).
 - Non-material surface condition at basal surface due to entrainment/deposition.

- **basic concept**



Erosion-deposition rate

- *BCRE-Model (1994), Tai & Kuo (Acta Mech., 2008), Tai & Lin (Phys. Fluids, 2008)*

Three states for basal surface:

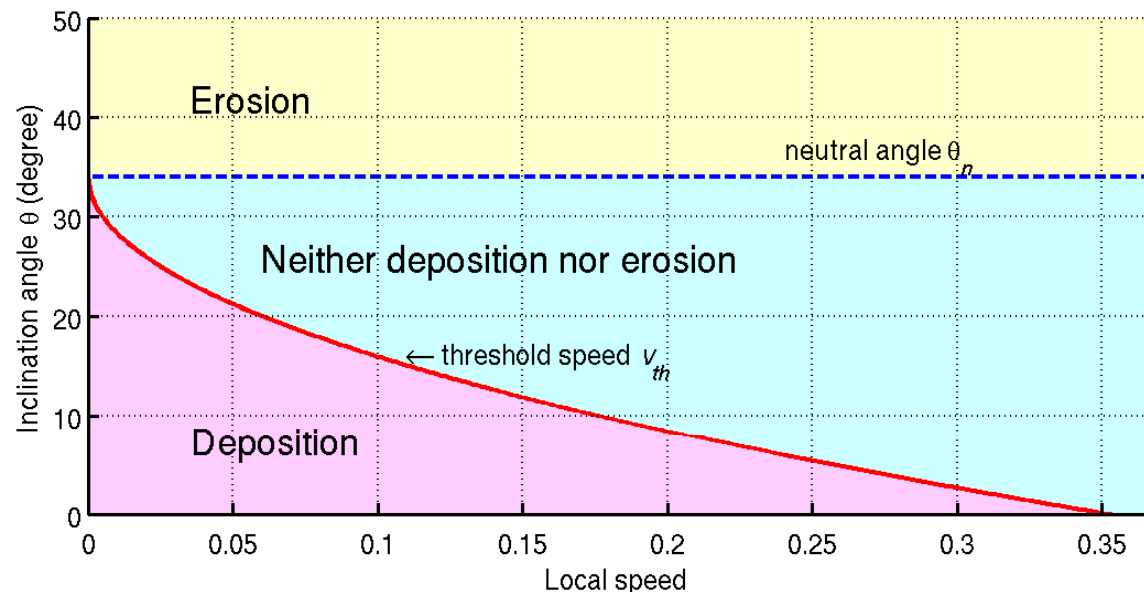
$$\begin{cases} \theta < \theta_n \text{ and } \|\mathbf{q}_{\parallel}\| > v_{th} \Rightarrow \mathcal{E}_+ = 0 : \text{immobile basal surface,} \\ \theta < \theta_n \text{ and } \|\mathbf{q}_{\parallel}\| < v_{th} \Rightarrow \mathcal{E}_+ > 0 : \text{deposition,} \\ \theta > \theta_n \Rightarrow \mathcal{E}_+ < 0 : \text{erosion.} \end{cases}$$

where

$$\theta = \sin^{-1} \left(\frac{-w}{\|\mathbf{q}_{\parallel}\|} \right)$$

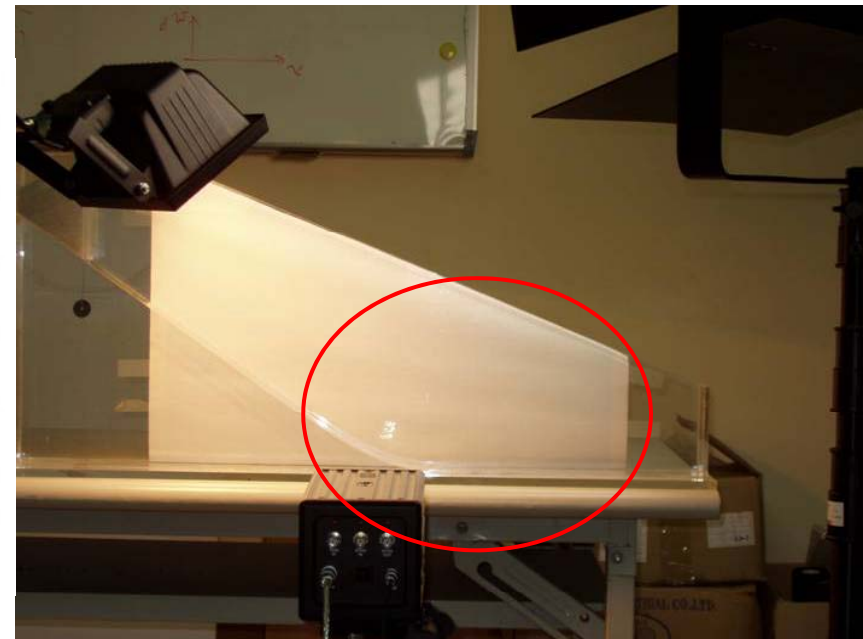
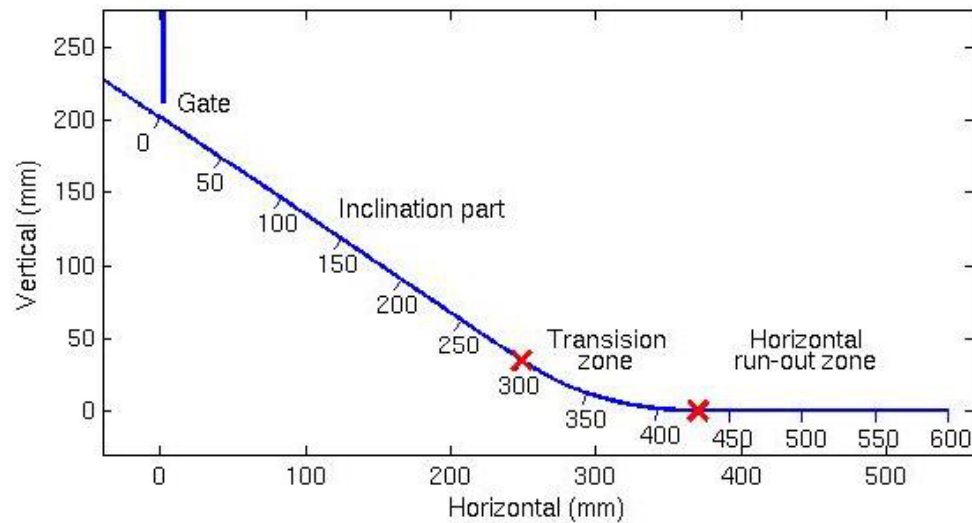
Remark:

any reasonable erosion rate can be applied to this proposed formulation.



Is the temporally variable coordinates necessary?

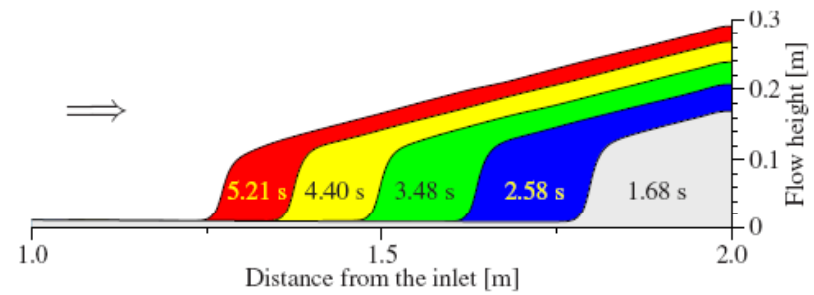
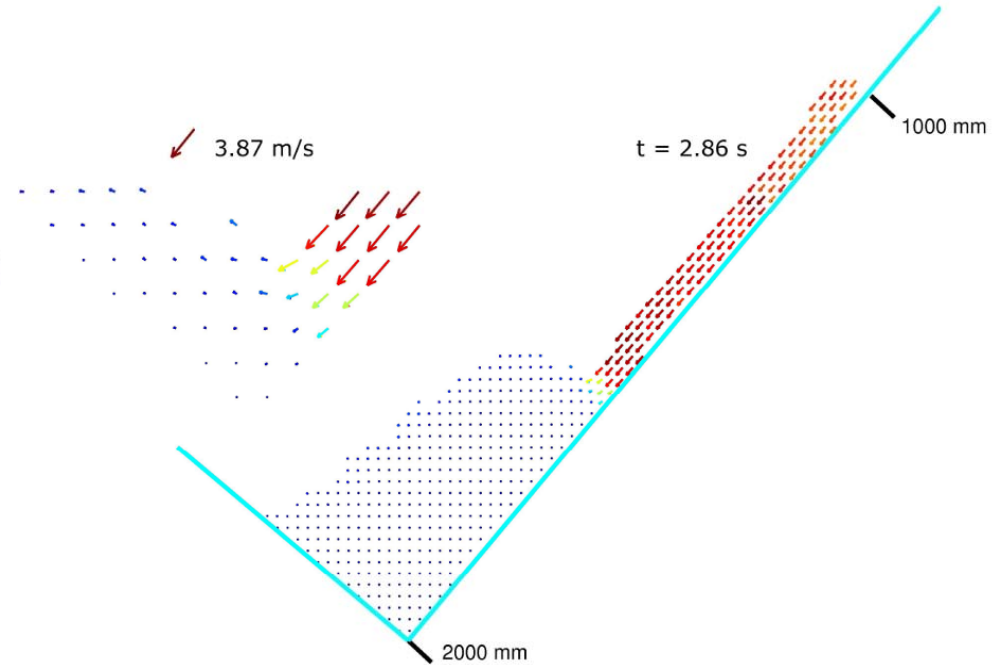
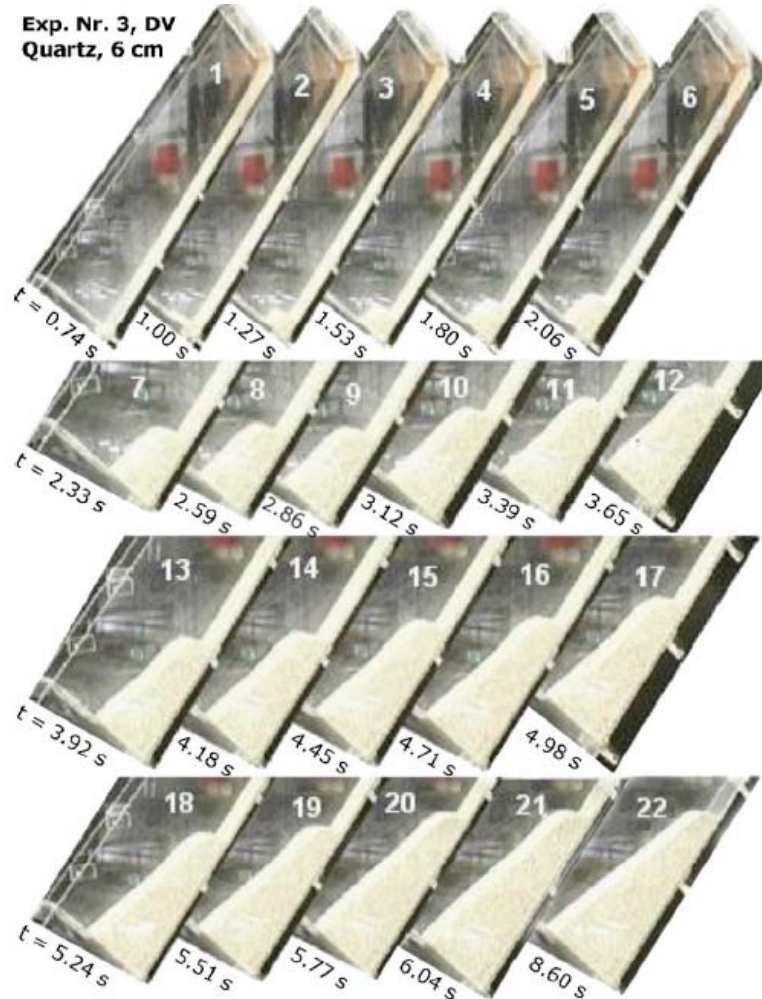
- Material : Ottawa sand (ϕ : 0.3~0.6 mm)
- Width of channel: 38 mm, Inclination angle: 34° to horizontal
- Gate opening at the top: 10, 15, 20 mm



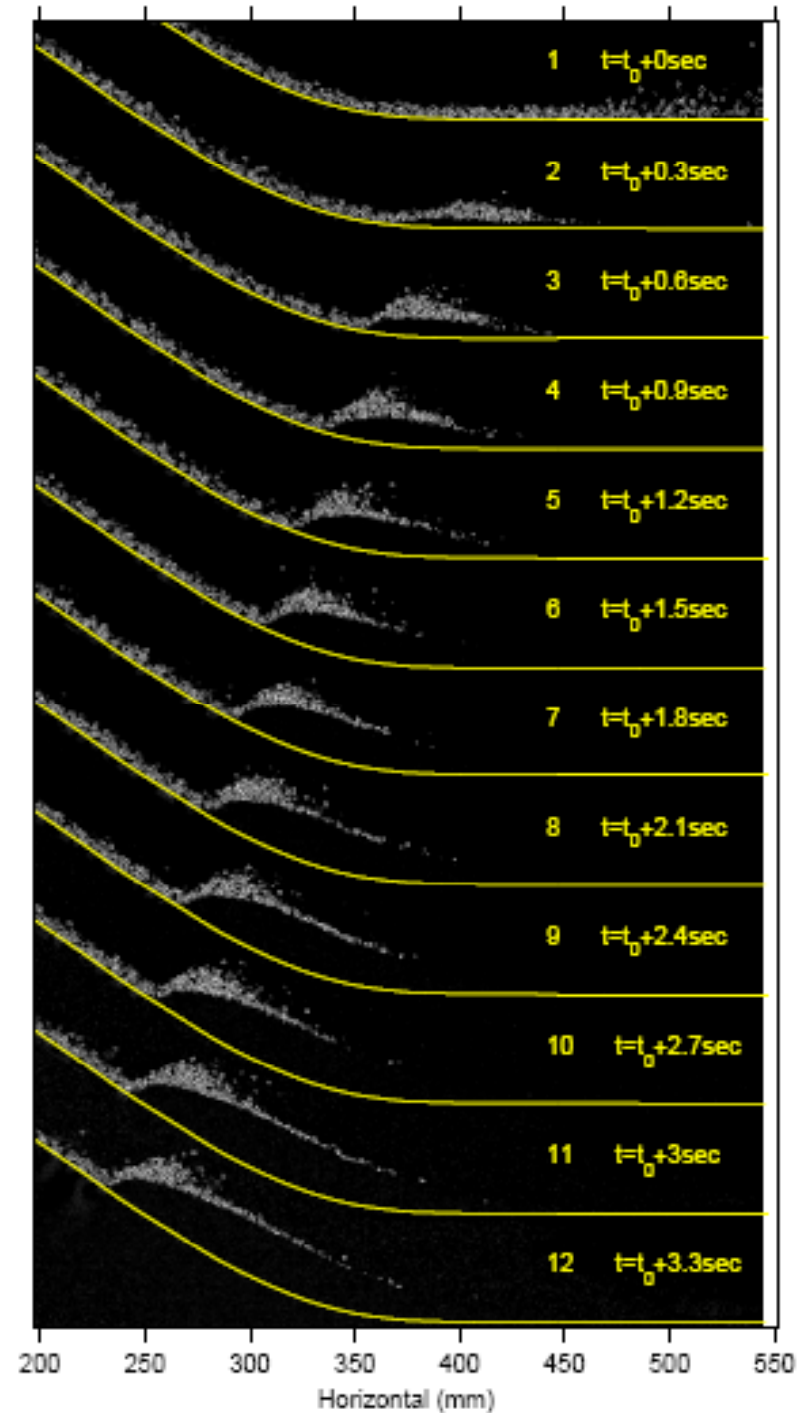
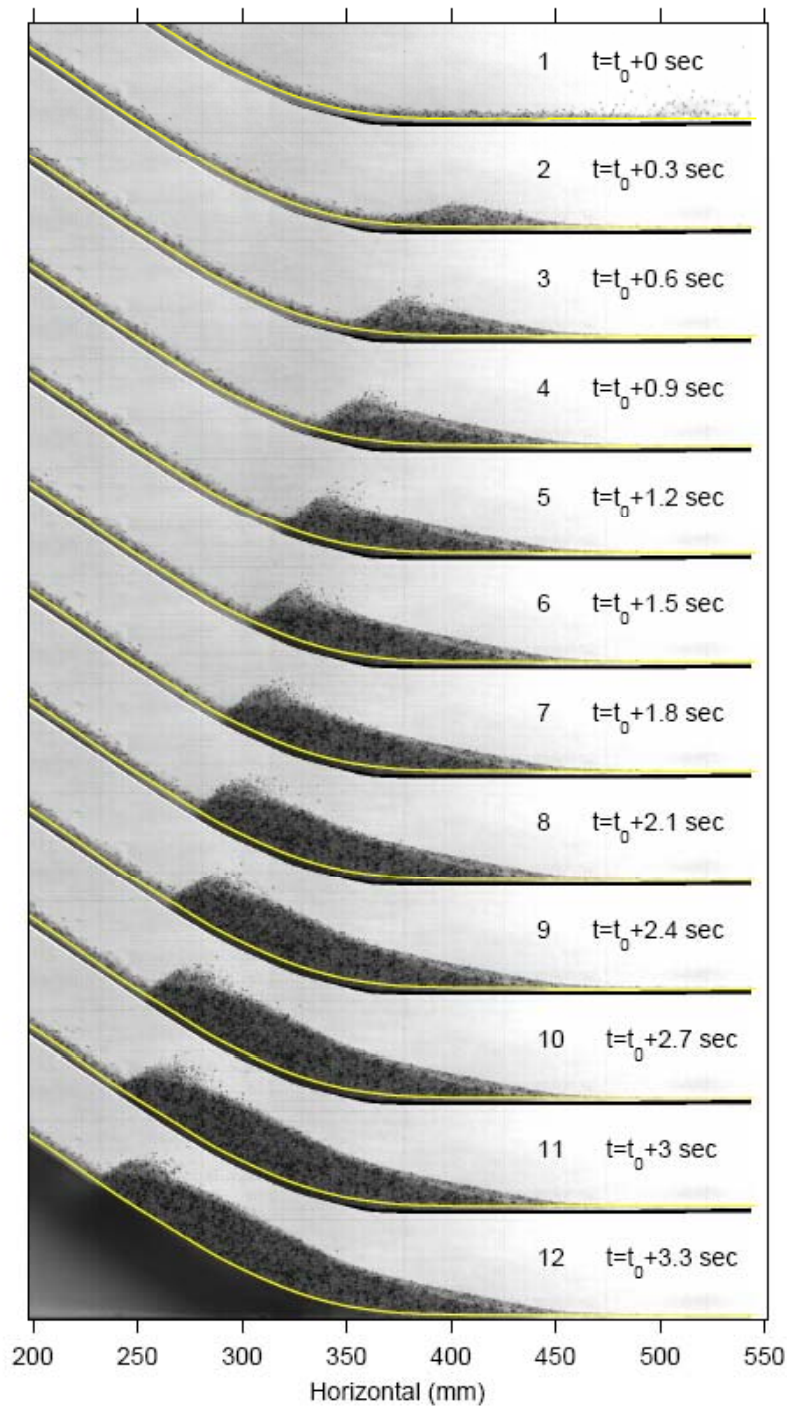
Tai & Lin (Phys. Fluids, 2008)

Traveling shock wave: Pudasaini et al, Phys. Fluids (2007)

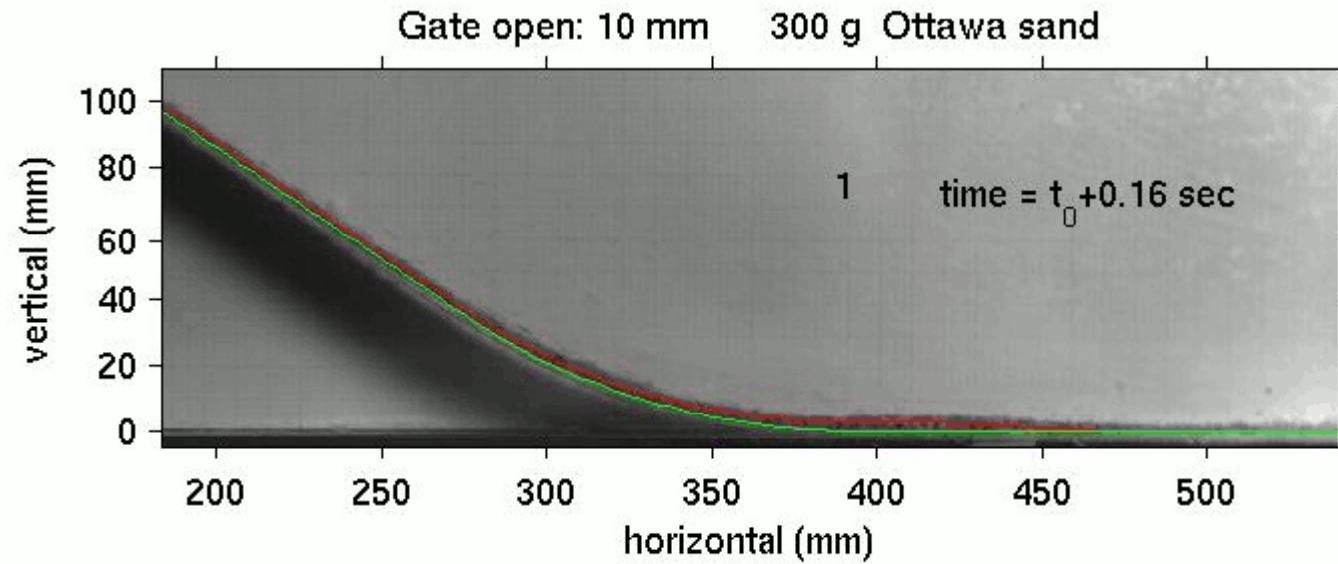
Exp. Nr. 3, DV
Quartz, 6 cm



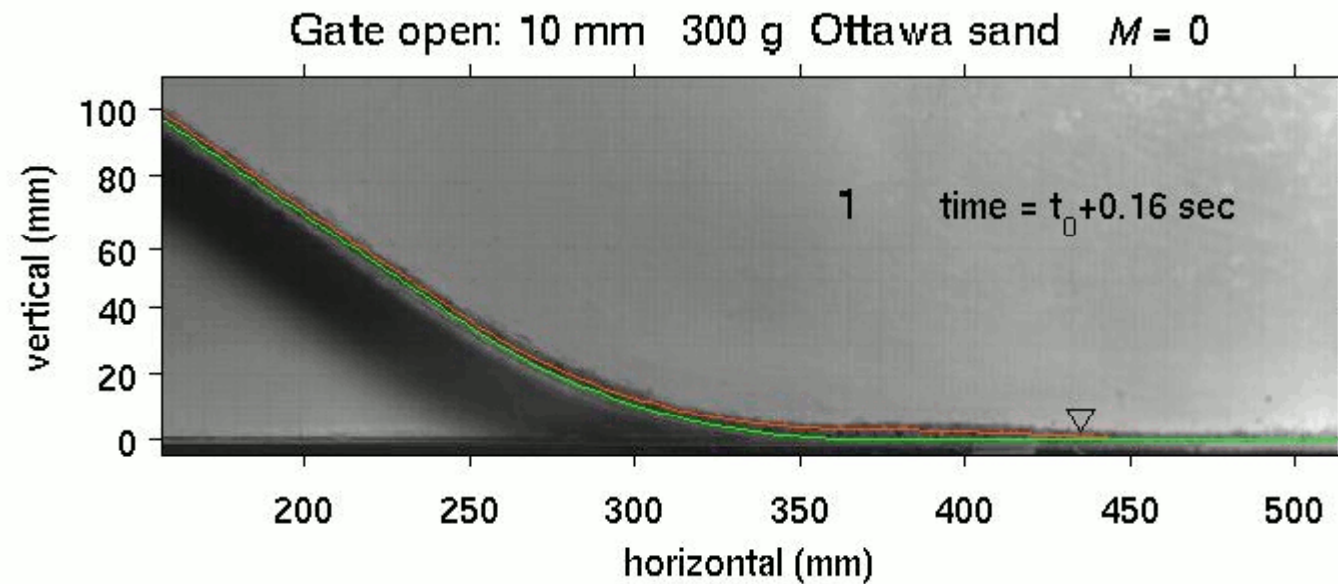
Pudasaini and Kröner, Phy. Rev. E (2008)



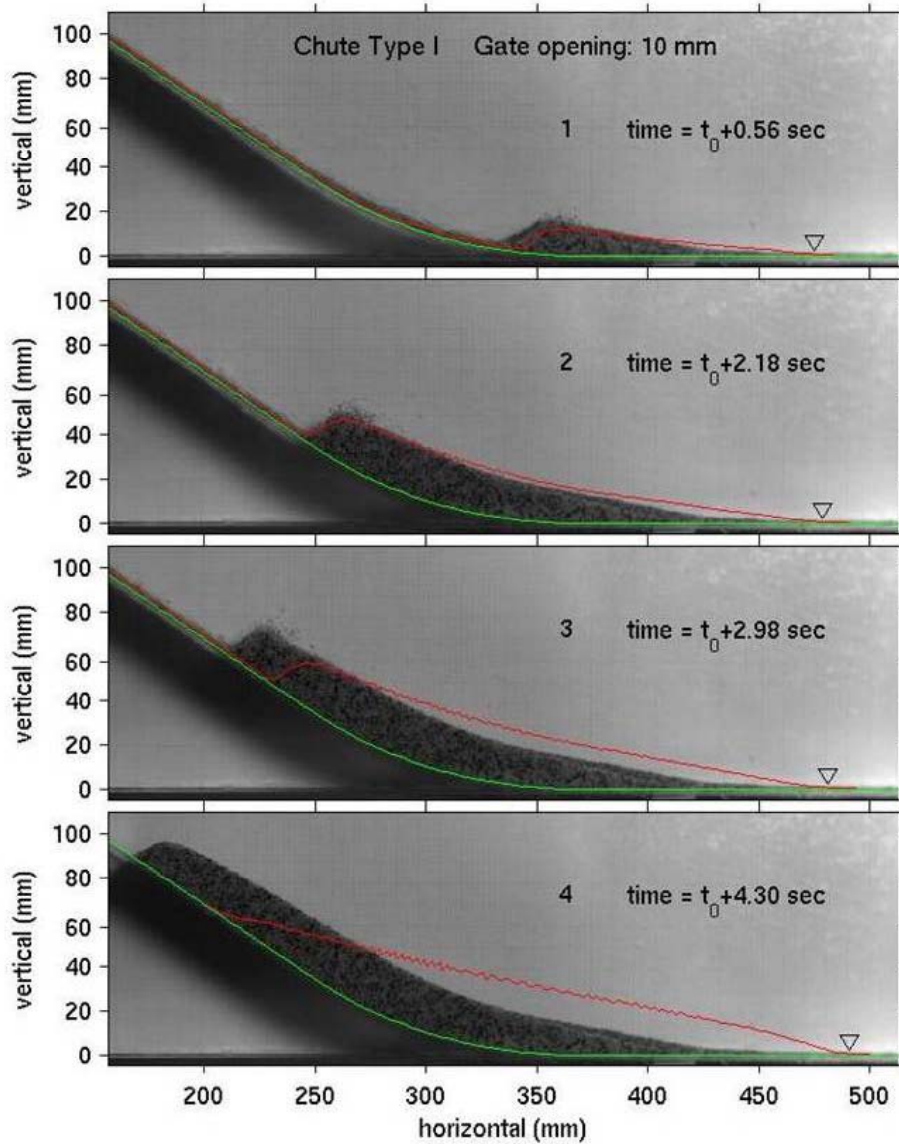
- Comparison with experimental data (*with deposition*)



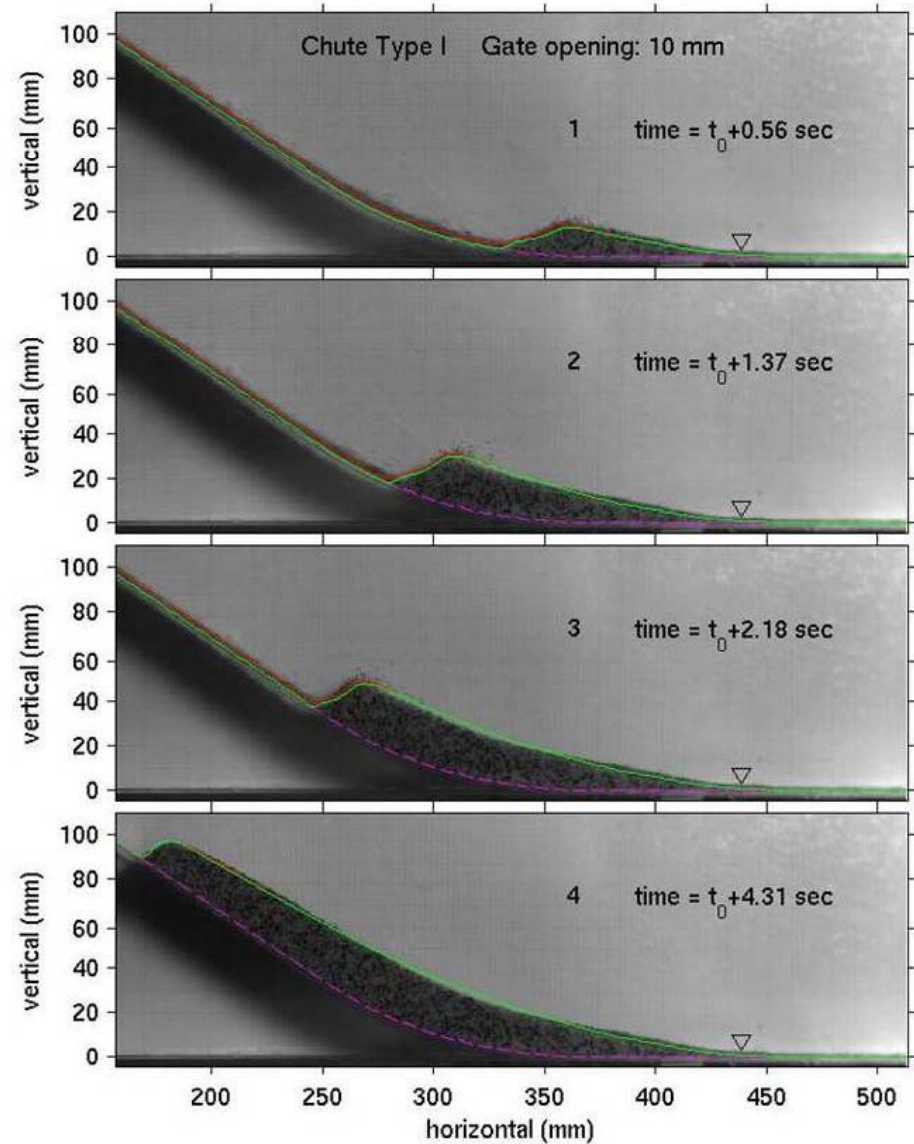
- Comparison with experimental data (*without deposition*)

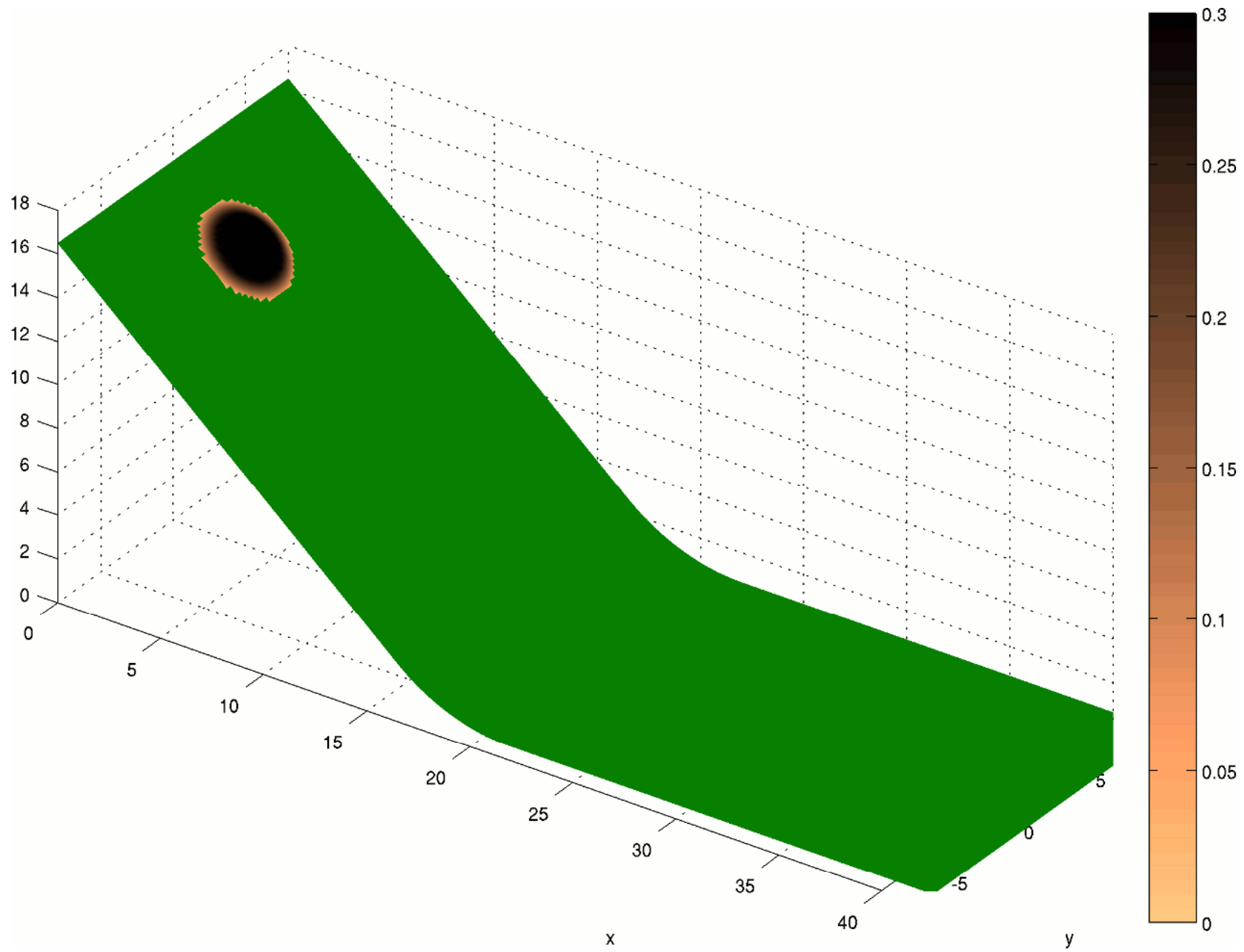


Without Deposition



With deposition



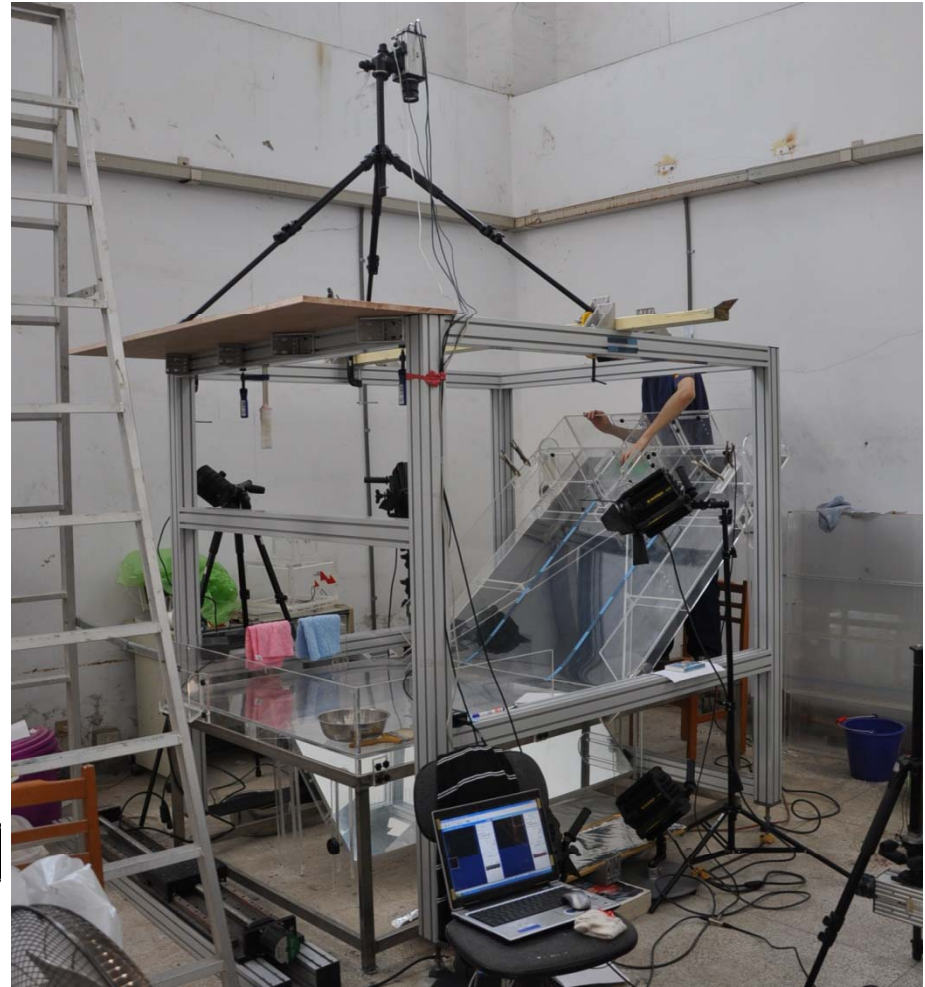
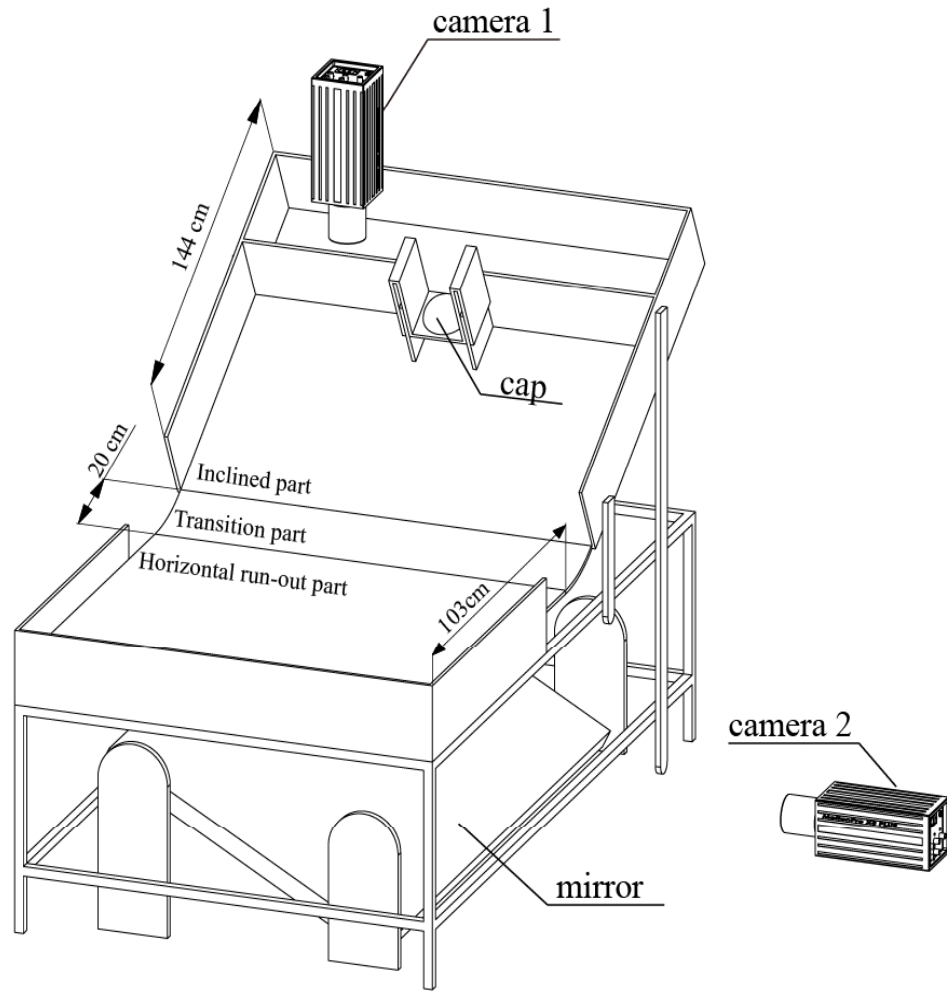


Experiments and comparison with theoretical prediction

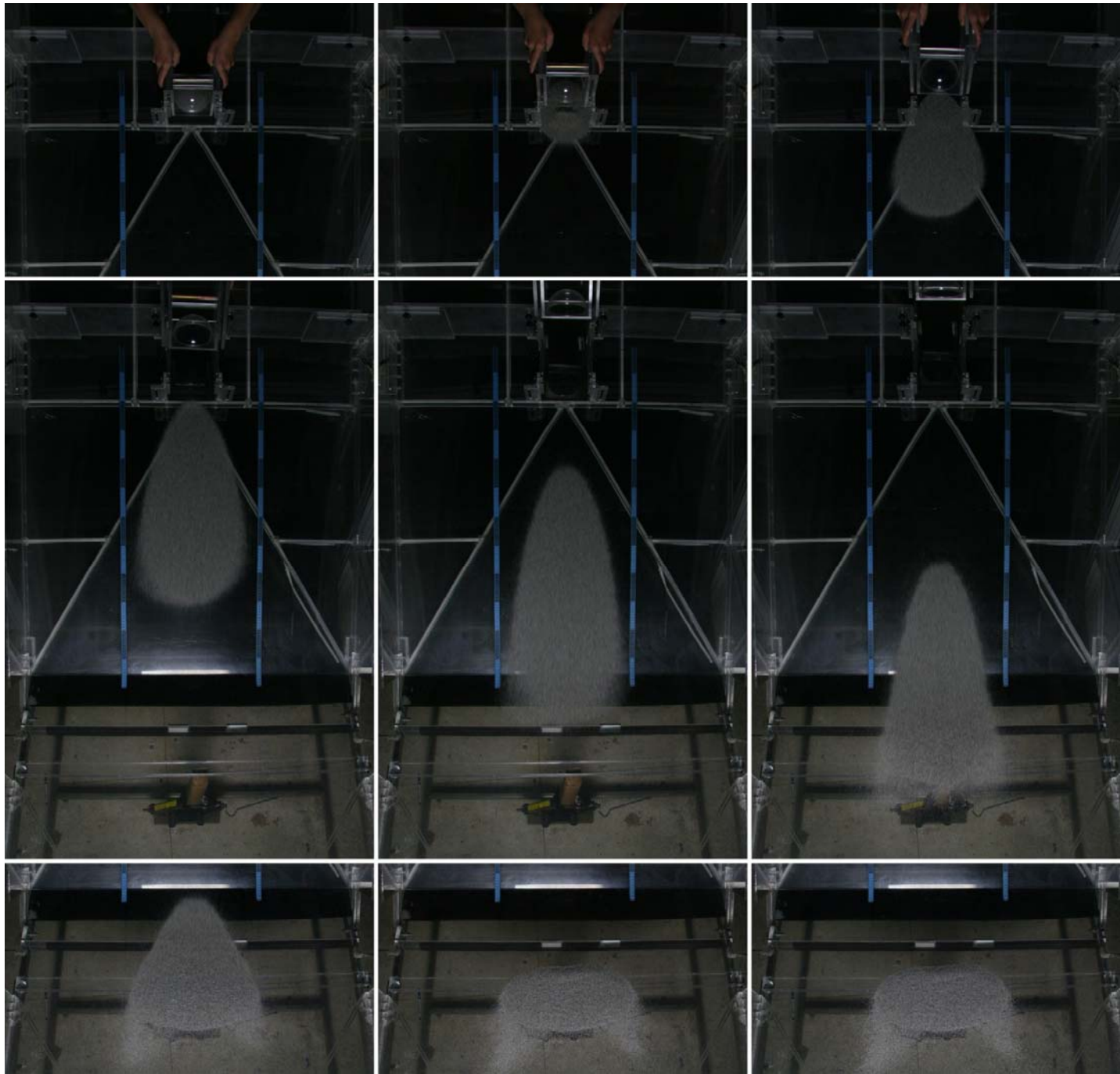
by

L.T. Sheng, Y.C. Tai, C. Y. Kuo and S.S. Hsiau

Experimental setup



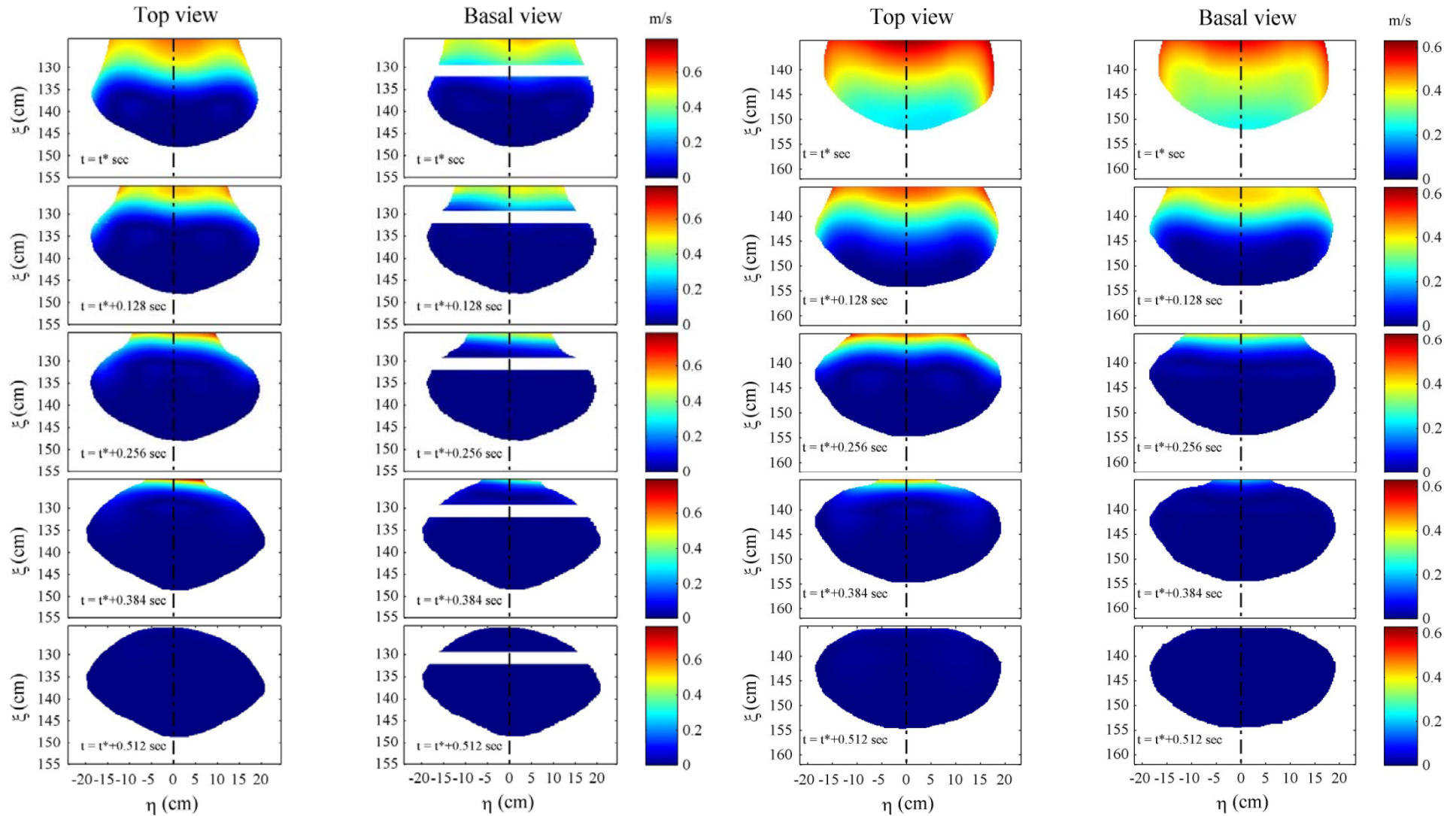
Three inclination angles: 32, 35 and 38 degree



Experimental results

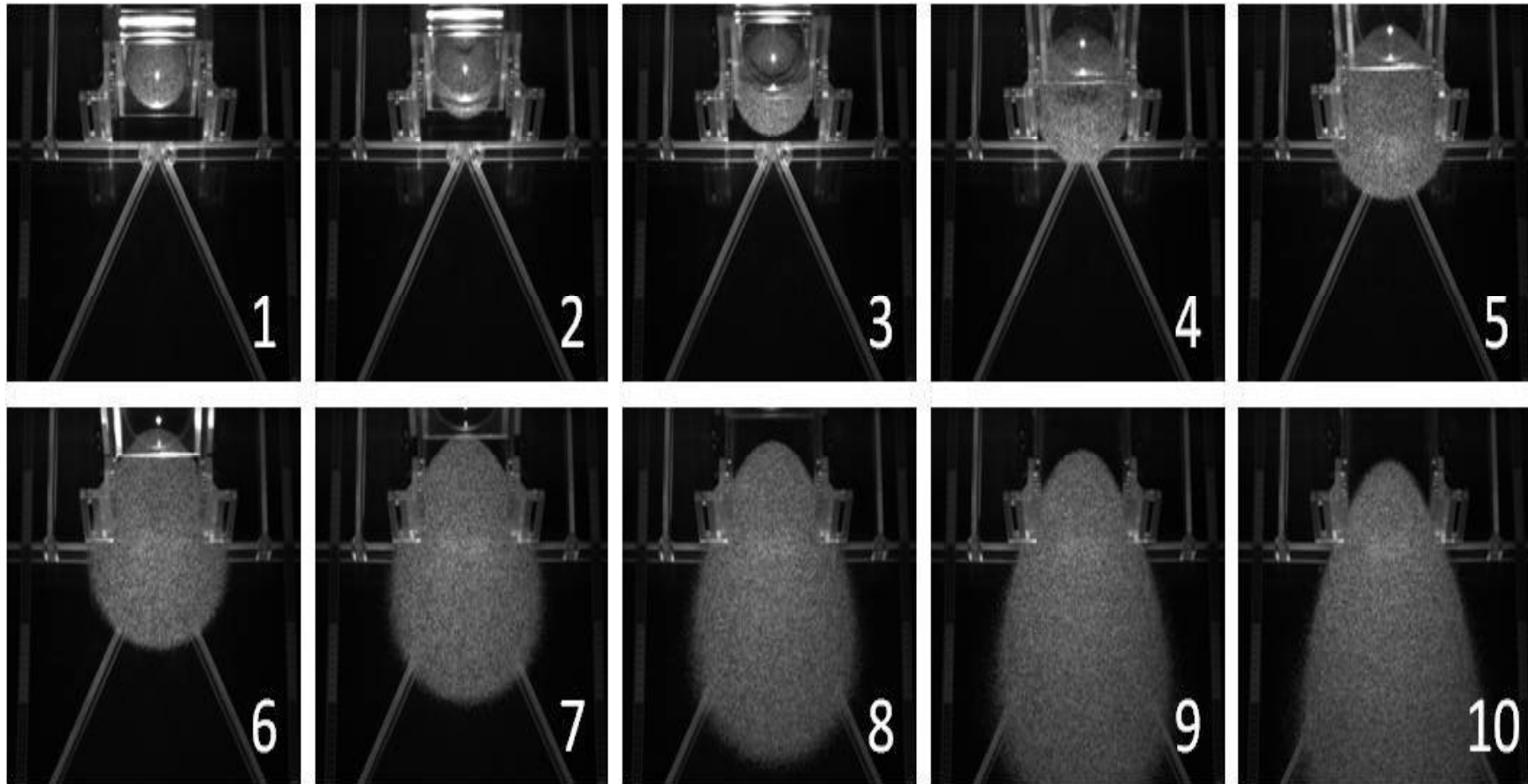
32 degree

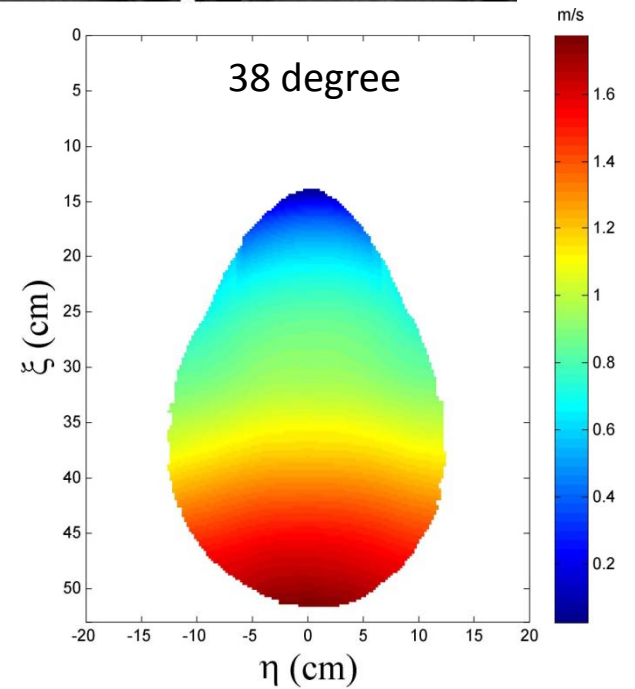
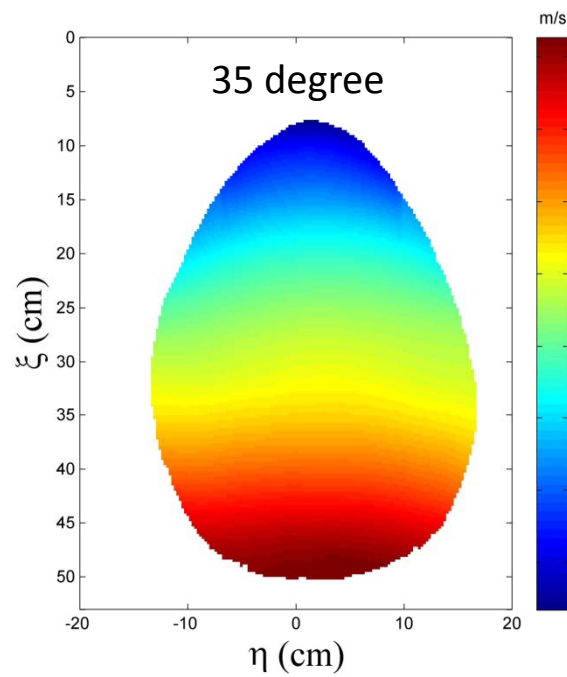
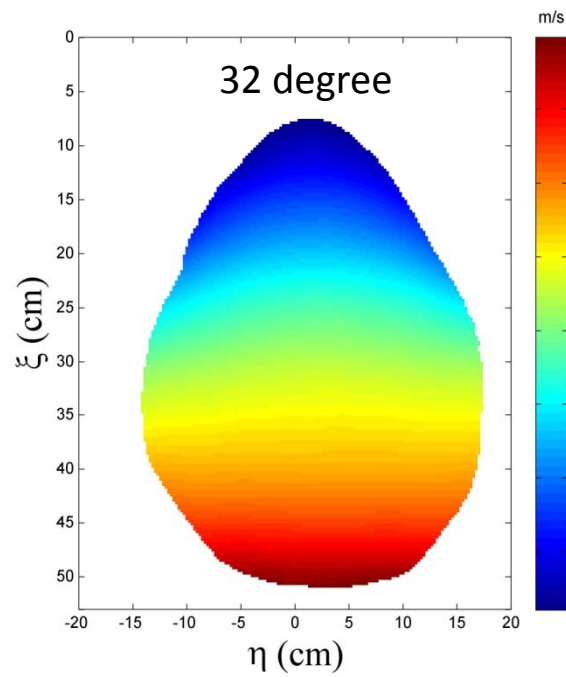
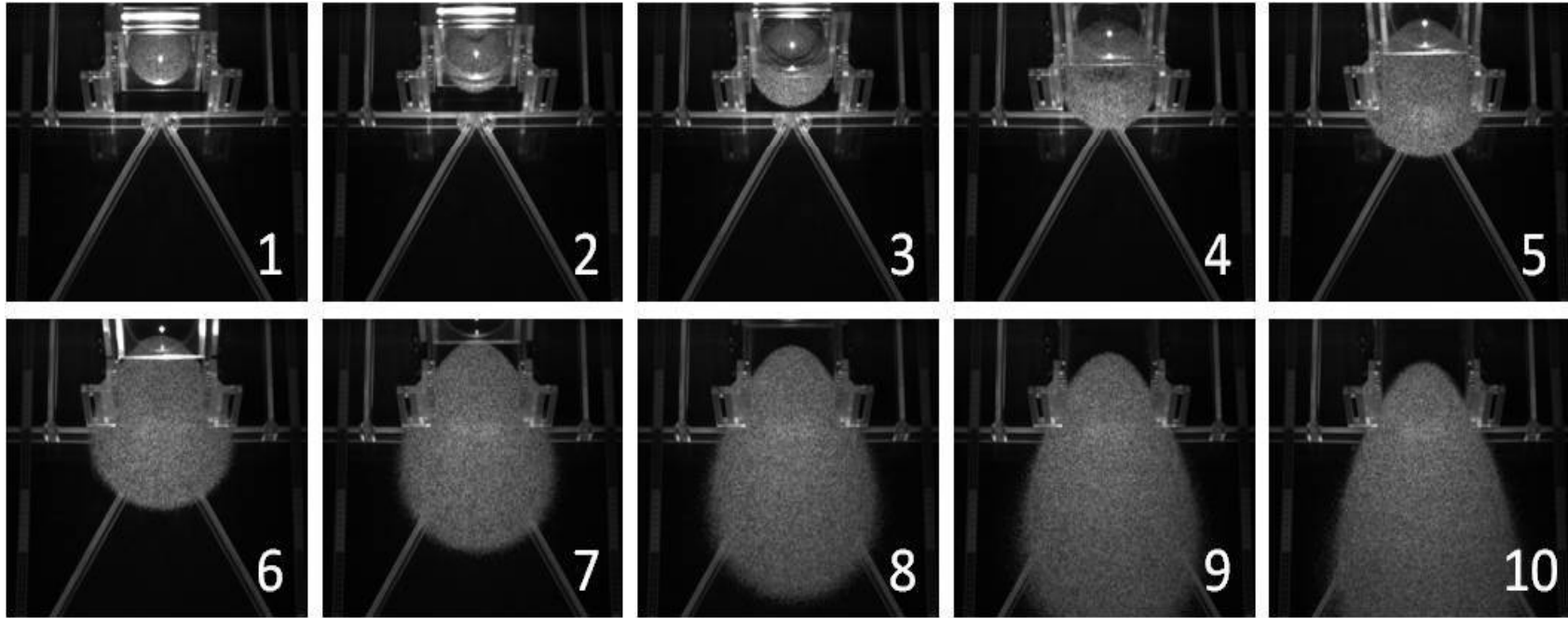
35 degree



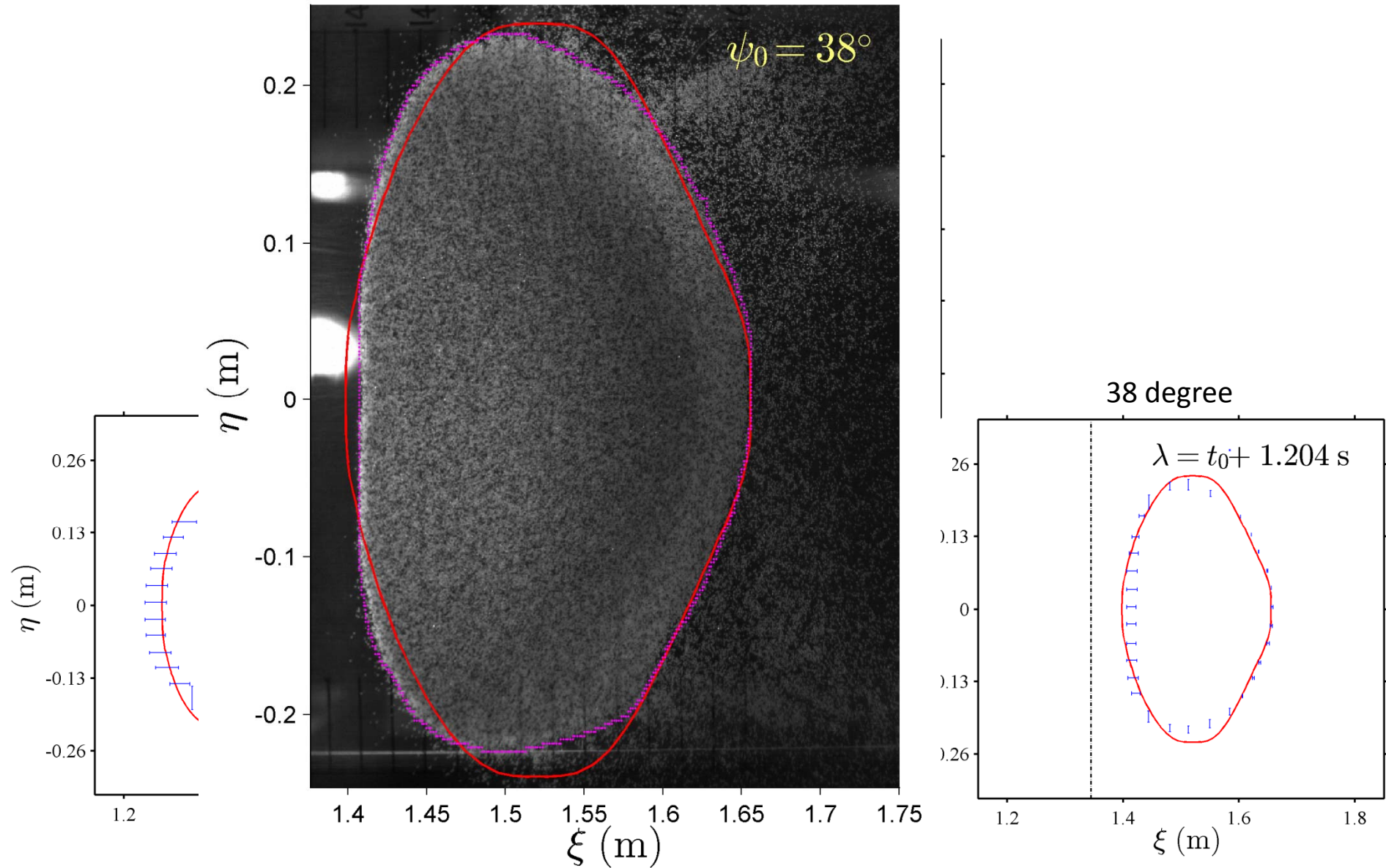
Experimental results

- Initial condition for theoretical prediction





Deposition position



Further applications

Coulomb-Mixture Theory (with deposition)

by

Y. C. Tai, and C. Y. Kuo, Nat. Hazards Earth Syst. Sci. (2012)

Coulomb-mixture theory (Iverson & Denlinger, JGR 2001)

- Conservation of mass and momentum

$$\begin{aligned} \frac{\partial \rho_s}{\partial t} + \operatorname{div}(\rho_s \vec{q}_s) &= 0, & \frac{\partial \rho_s \vec{q}_s}{\partial t} + \operatorname{div}(\rho_s \vec{q}_s \otimes \vec{q}_s - \tilde{\mathbf{T}}_s) &= \rho_s \vec{g} + \vec{m}, \\ \frac{\partial \rho_f}{\partial t} + \operatorname{div}(\rho_f \vec{q}_f) &= 0, & \frac{\partial \rho_f \vec{q}_f}{\partial t} + \operatorname{div}(\rho_f \vec{q}_f \otimes \vec{q}_f - \tilde{\mathbf{T}}_f) &= \rho_f \vec{g} - \vec{m}, \end{aligned}$$

- Transformation into UC system

$$\begin{aligned} \partial_\lambda(J\nu_s) + \partial_m[J\nu_s(q_s^m - Q^m)] &= 0, & \partial_\lambda(J\nu_f) + \partial_m[J\nu_f(q_f^m - Q^m)] &= 0, \\ \partial_\lambda(J\nu_s u_s^i) + \partial_m[J\nu_s u_s^i(q_s^m - Q^m) - J\Sigma_s^{im}] &= J\nu_s g^i + \frac{1}{\hat{\rho}_s} m^i, \\ \partial_\lambda(J\nu_f u_f^i) + \partial_m[J\nu_f u_f^i(q_f^m - Q^m) - J\Sigma_f^{im}] &= J\nu_f g^i - \frac{1}{\hat{\rho}_f} m^i, \end{aligned}$$

where

$$i \in \{x, y, z\} \text{ and } m \in \{\xi, \eta, \zeta\}$$

$$\rho_f = \nu_f \hat{\rho}_f \quad \rho_s = \nu_s \hat{\rho}_s$$

$$\tilde{\mathbf{T}}_{s,f} = T_{s,f}^{ij} \hat{\mathbf{e}}_i \otimes \hat{\mathbf{e}}_j = \Sigma_{s,f}^{im} \hat{\mathbf{e}}_i \otimes \vec{\mathbf{g}}_m$$

$$\hat{\rho}_s = \text{const. and } \hat{\rho}_f = \text{const.}$$

Coulomb-mixture theory (Tai & Kuo, Nat. Hazards Earth Syst. Sci., 2012)

- **Assumptions in Coulomb mixture theory**

(a) Shallowness of the flowing layer and *the curvature*

(b) Saturation: $\nu_s + \nu_f = 1$

(c) Small relative velocity: $|\vec{q}_f - \vec{q}_s| \ll |\vec{q}_s| \Rightarrow \vec{m} = 0$

(d) Erosion/deposition rates of both solid and fluid constituents are equal 

(e) Total stress: $\tilde{\mathbf{T}} = \tilde{\mathbf{T}}_s + \tilde{\mathbf{T}}_f$ with $\tilde{\mathbf{T}}_s = \tilde{\mathbf{T}}_{eff}$ and $\tilde{\mathbf{T}}_f = -\mathbf{I}p + \nu_f \tilde{\mathbf{T}}_{vis}$

(f) Solid constituent: Coulomb friction at the basal surface

(g) Fluid constituent: Newtonian fluid

(h) Earth pressure coefficient: $K_{act/pas} = 1$ 

- *Depth-integration with boundary conditions (BCs)*

Coulomb-mixture theory (Tai & Kuo, Nat. Hazards Earth Syst. Sci., 2012)

- The leading-order governing equations of Coulomb mixture theory in UC system

$$\begin{aligned} \frac{\partial}{\partial \lambda}(J_b h) + \frac{\partial}{\partial \xi}(J_b h q_0^\xi) + \frac{\partial}{\partial \eta}(J_b h q_0^\eta) &= -J_b \mathcal{E}_- + \mathcal{O}(\epsilon^{1+\gamma}). \\ \frac{\partial}{\partial \lambda}(J_b h u_0^i) + \frac{\partial}{\partial \xi}(J_b h u_0^i q_0^\xi) + \frac{\partial}{\partial \eta}(J_b h u_0^i q_0^\eta) - \frac{\partial}{\partial \xi} \left((1 - \Lambda_f) \epsilon J_b h \overline{\Sigma^{i\xi}} \right) - \frac{\partial}{\partial \eta} \left((1 - \Lambda_f) \epsilon J_b h \overline{\Sigma^{i\eta}} \right) \\ &= -J_b \mathcal{E}_- u_b^i - \epsilon^\alpha J_b h A_c n^i + J_b h c n^i - (1 - \Lambda_f) \epsilon^\beta J_b N_b \mu \frac{u_b^i}{\|\vec{q}_b\|} + J_b s_f^i + J_b h g^i + \mathcal{O}(\epsilon^{1+\gamma}) \end{aligned}$$

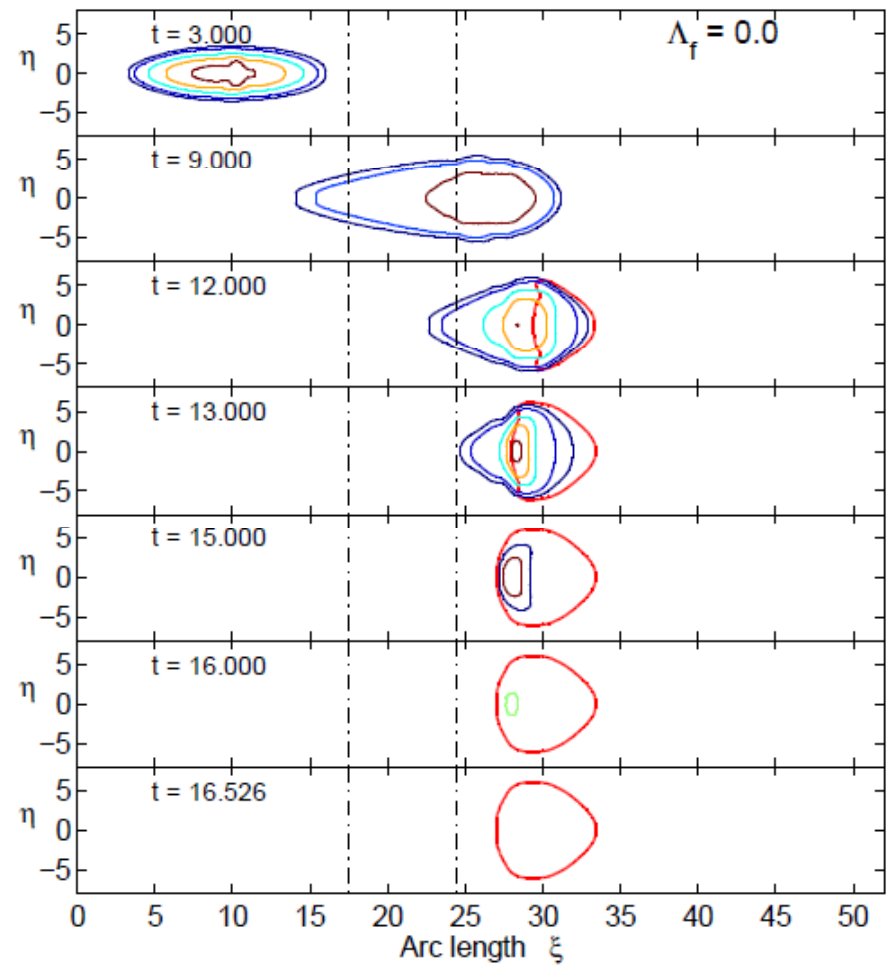
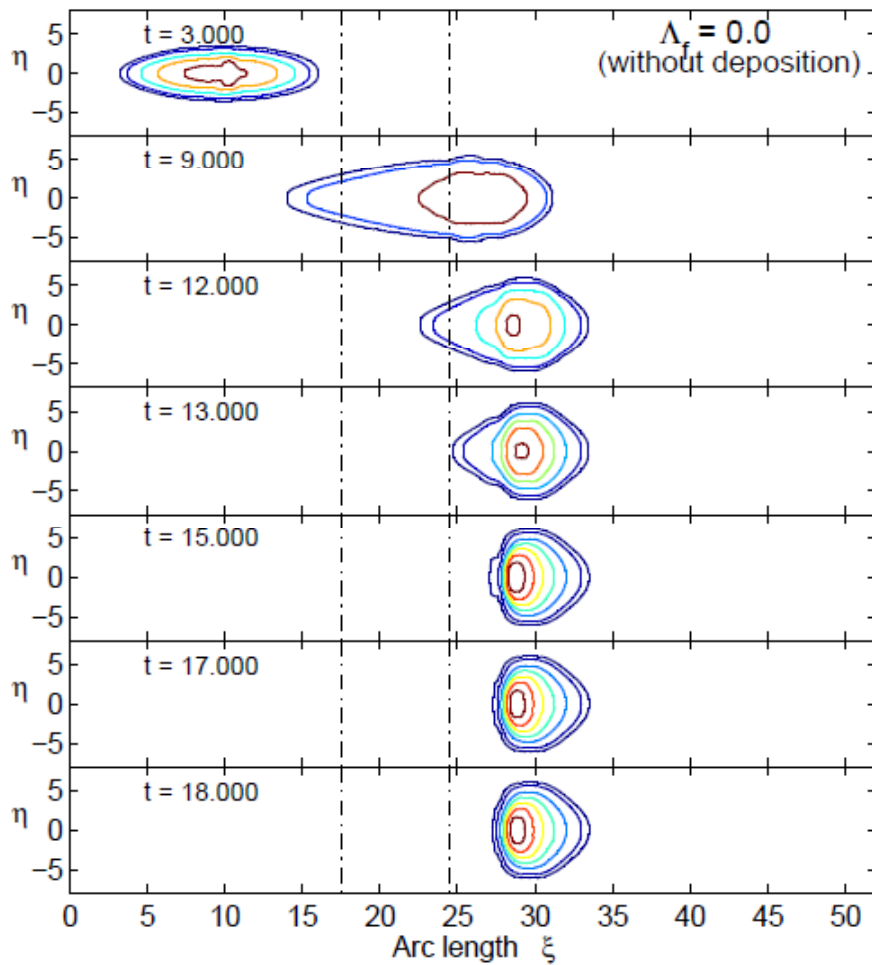
with

$$s_f^i = \tilde{\Omega}_m^i \left(-\epsilon h \frac{\partial(p_{bed})}{\partial m} + \epsilon \frac{\nu_f h}{N_R} \frac{\partial^2(q_0^m)}{\partial n^2} - \frac{\nu_f}{\epsilon} \frac{3}{N_R} \frac{q_0^m}{h} \right) \quad \text{and} \quad p_{bed} = \Lambda_f h c$$

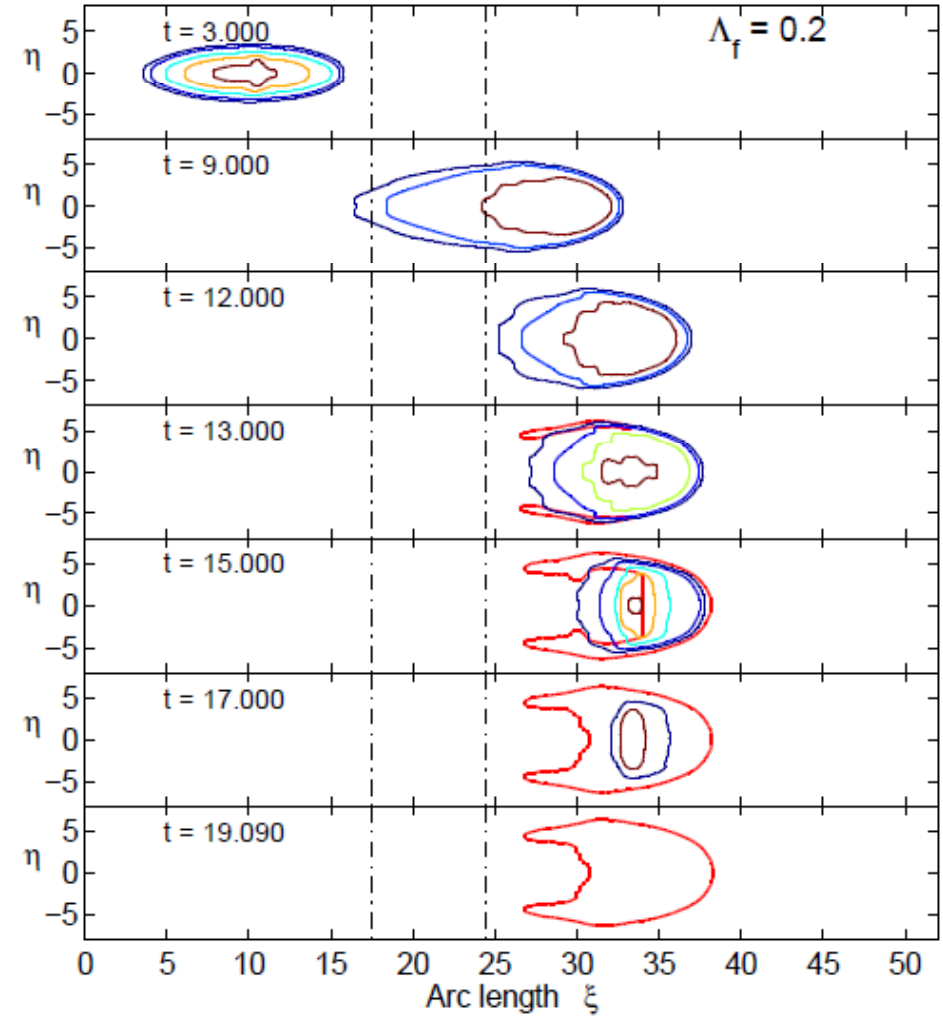
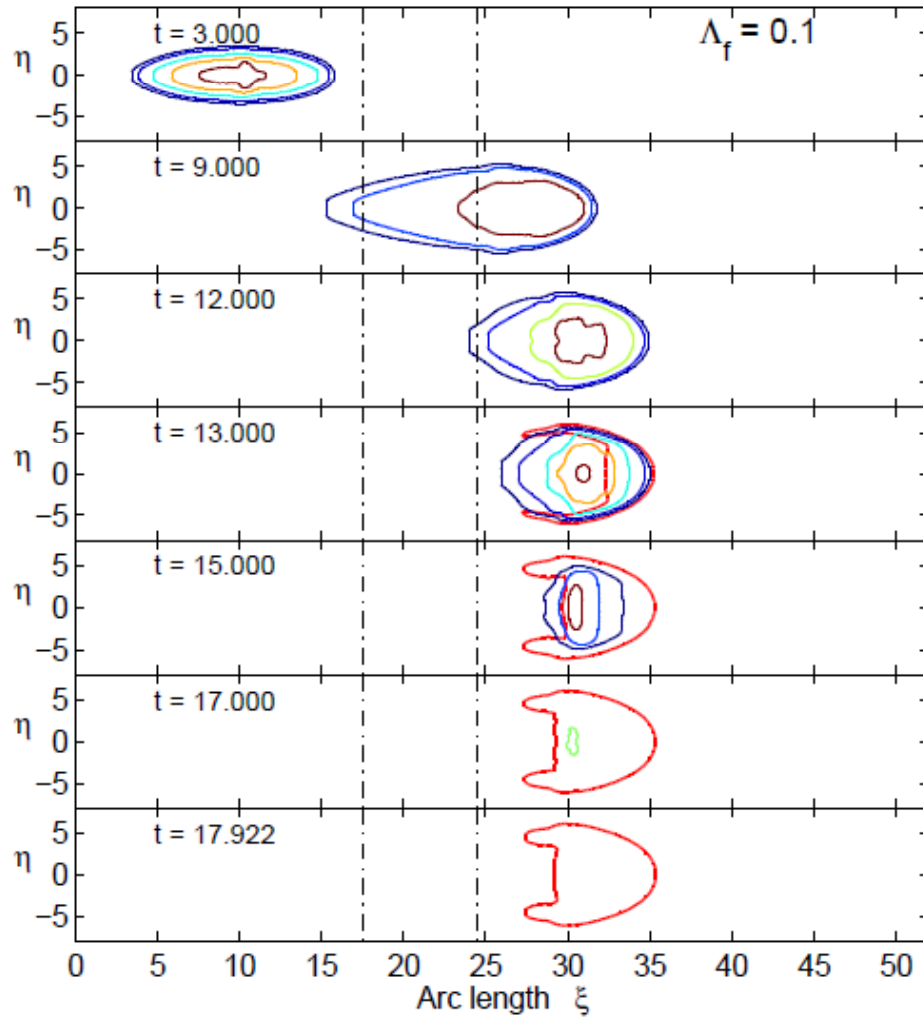
$N_R = \rho H \sqrt{gL} / \mu$ is analogous to the Reynolds number for Newtonian fluid.

$$i \in \{x, y\}, \quad m \in \{\xi, \eta\}$$

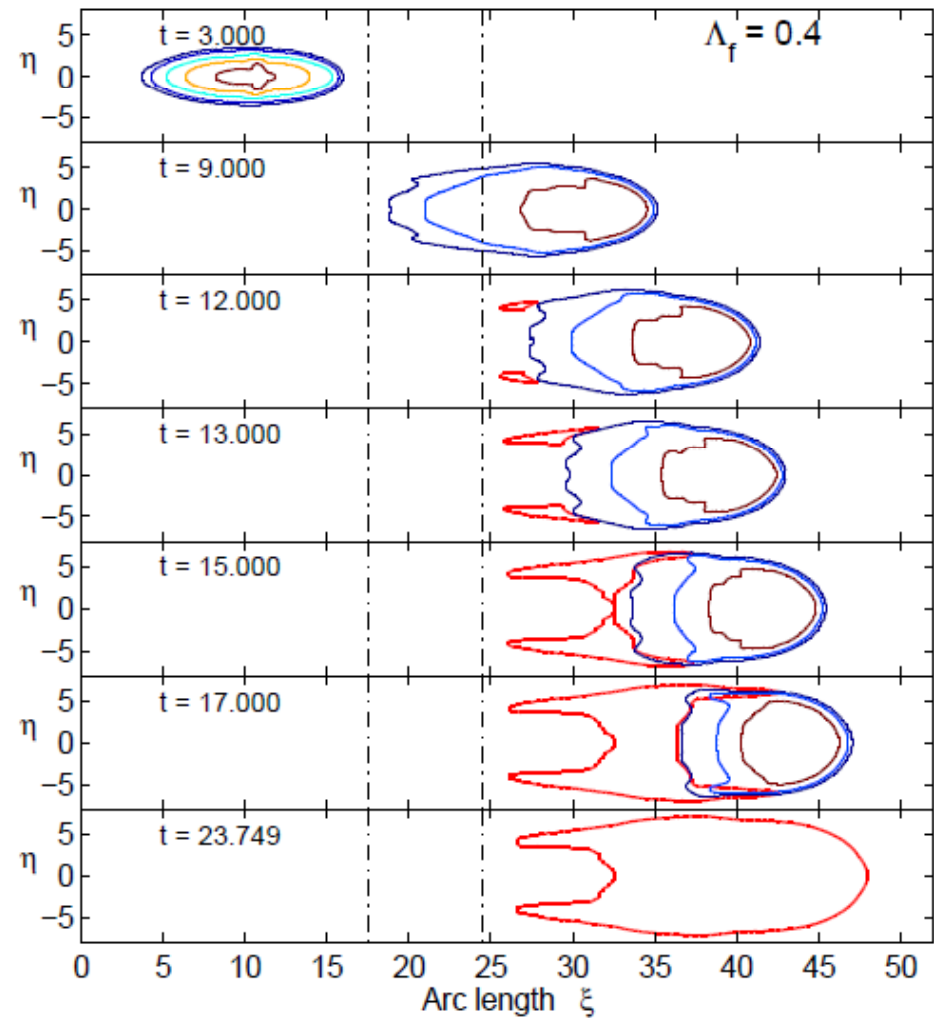
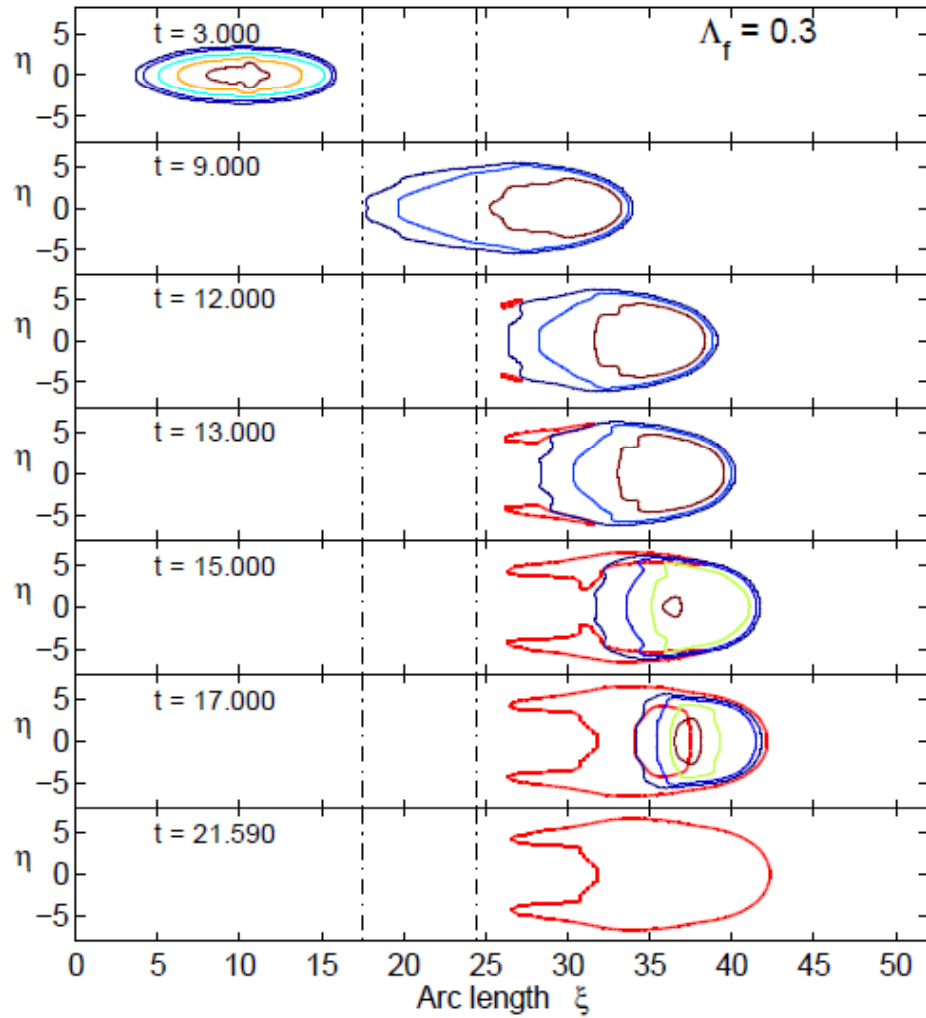
Comparison without/with deposition procedure



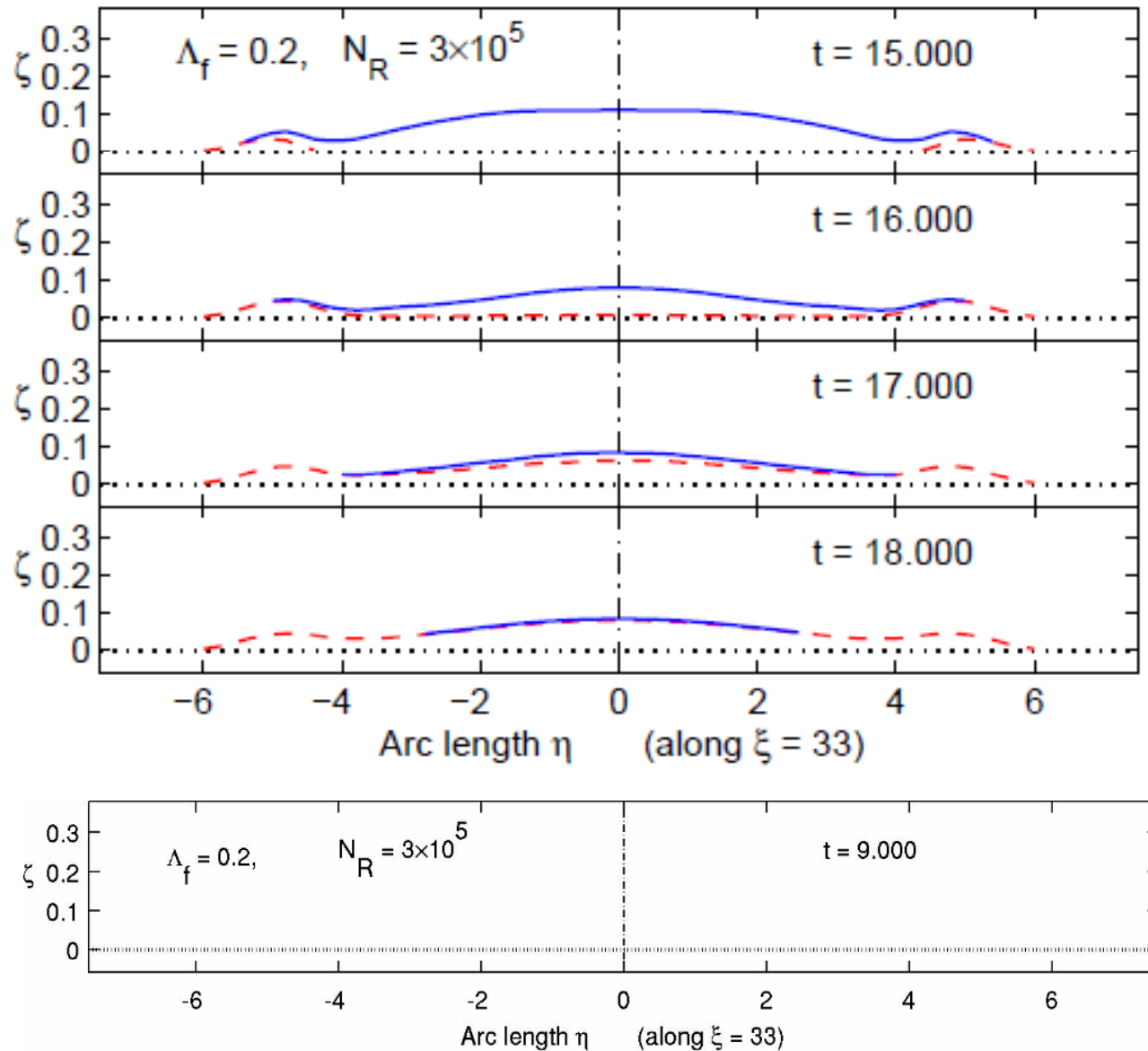
Levee formation



Levee formation



Levee formation (cross section view)



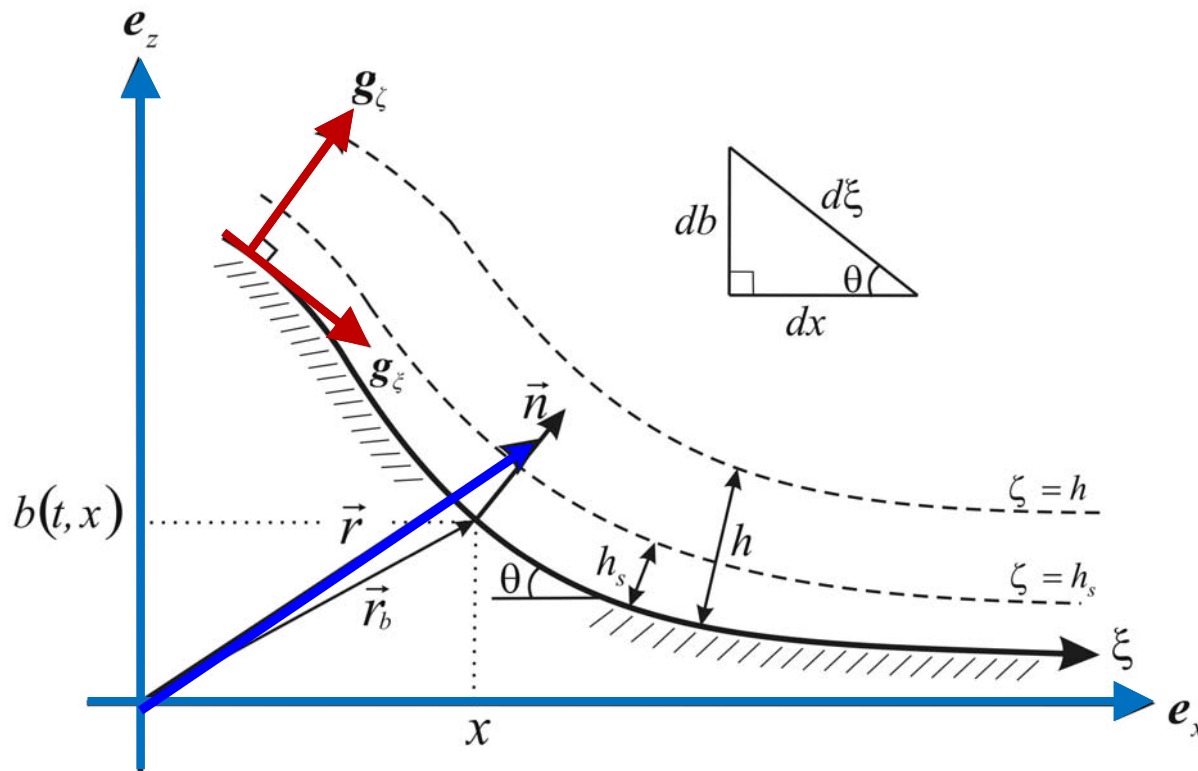
Further applications

Erosional Dam-Break Shear Flows

by

Y. T. Huang and Y. C. Tai

Coordinate system for general topography

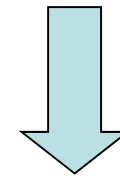


$$z - b(x, t) = 0$$

$$\vec{n} = -s_x \mathbf{e}_x + c \mathbf{e}_z$$

$$s_x = c \partial_x b$$

$$c = \frac{1}{\sqrt{1 + (\partial_x b)^2}}$$



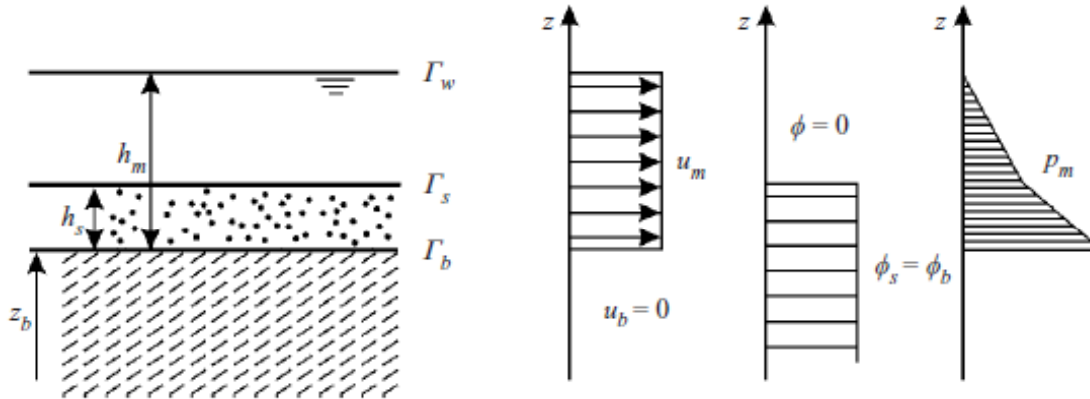
$$\vec{r} \equiv r_x \mathbf{e}_x + r_z \mathbf{e}_z = \vec{r}_b + \zeta \vec{n}$$

$$= x \mathbf{e}_x + b \mathbf{e}_z + \zeta \vec{n}$$

$$= (x - \zeta s_x) \mathbf{e}_x + (b + \zeta c) \mathbf{e}_z$$

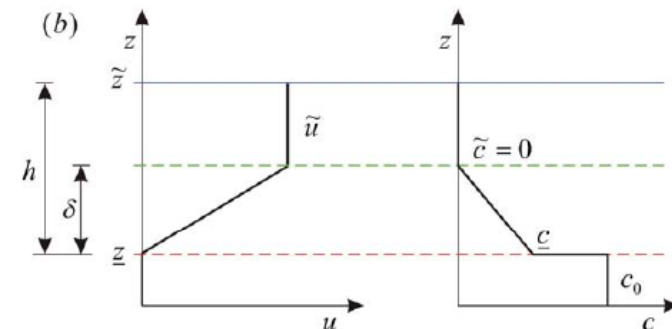
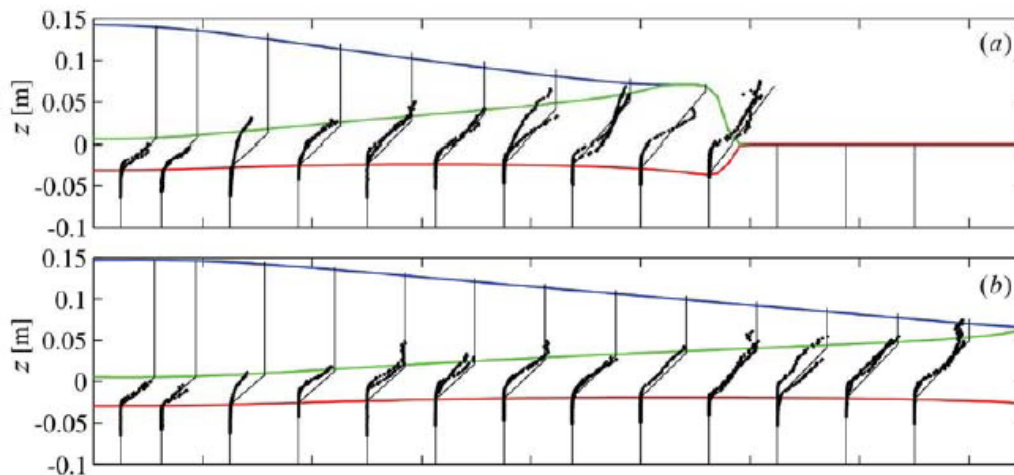
Flow structure postulation

- **Previous postulated flow structure** (Fraccarollo and Capart, 2002, JFM)



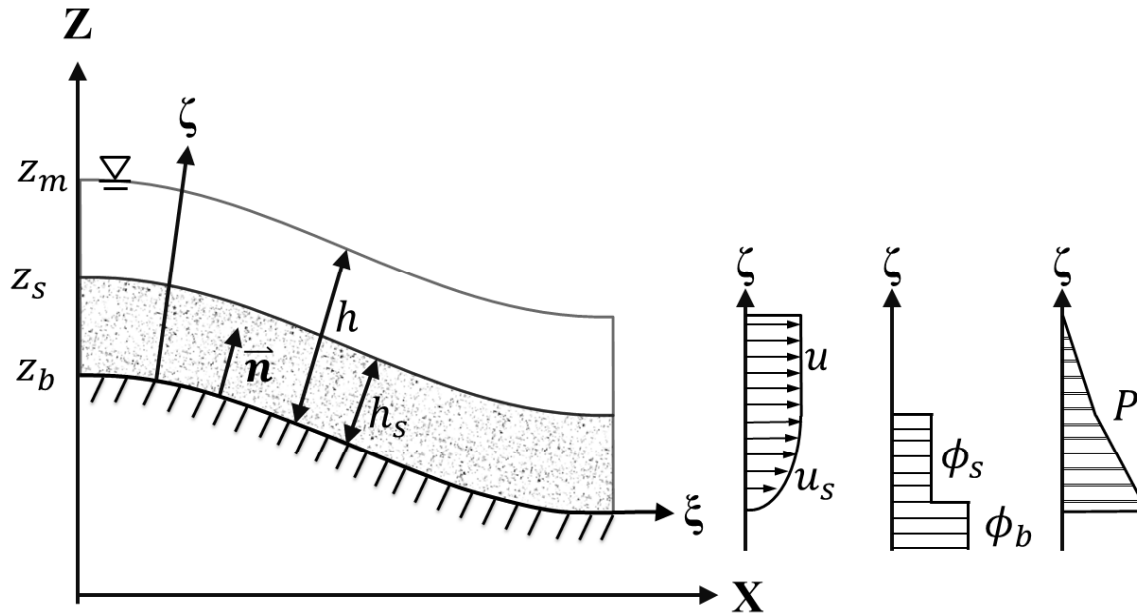
- thin layers
- almost uniform velocity profile
- constant sediment concentration (mixture layer and bed)
- hydrostatic pressure

- **Measurement of a sudden dam break** (Spinewine and Capart, 2013, JFM)



Flow structure postulation

- *Current postulated flow structure over the terrain-fitted coordinate system*



- *thin layers*
- *almost uniform velocity profile for water layer and linear velocity profile for mixture layer*
- *constant sediment concentration for mixture layer and bed but of different values*
- *hydrostatic distribution for the pressure*

Depth-integrated equations

- *Equations of mass balance*

$$\partial_t (J_b h) + \partial_\xi (J_b u^\xi (h - \frac{1}{2} h_s)) = -J_b \mathbf{E}$$

$$\partial_t (J_b h_s) + \partial_\xi (\frac{1}{2} J_b h_s u^\xi) = -J_b \alpha_C^{-1} \mathbf{E}$$

$$\alpha_C = \frac{\phi_s}{\phi_b}, \quad r = (\rho_s / \rho_w - 1) \phi_s$$

$$\Sigma^{x\xi} = \Omega_m^x t^{m\xi}, \quad \Sigma^{x\zeta} = \Omega_m^x t_b^{m\zeta}$$

$$J_b = \det \tilde{\Omega}_b$$

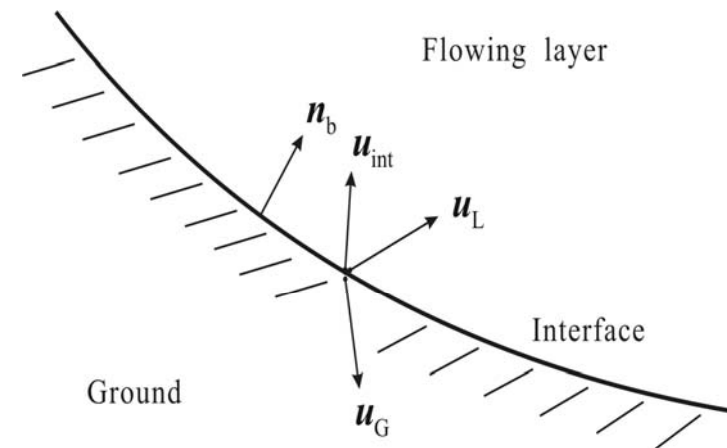
- *Equations of momentum balance*

$$\begin{aligned} \partial_t (J_b (h + \frac{r-1}{2} h_s) u^x) + \partial_\xi (J_b [(h + \frac{r-2}{3} h_s) u^x u^\xi + \Sigma^{x\xi}]) \\ = -J_b \mathbf{E} (h + r h_s) u_b^x - J_b s_x c (h + r h_s) g - J_b \Sigma_b^{x\zeta} \end{aligned}$$

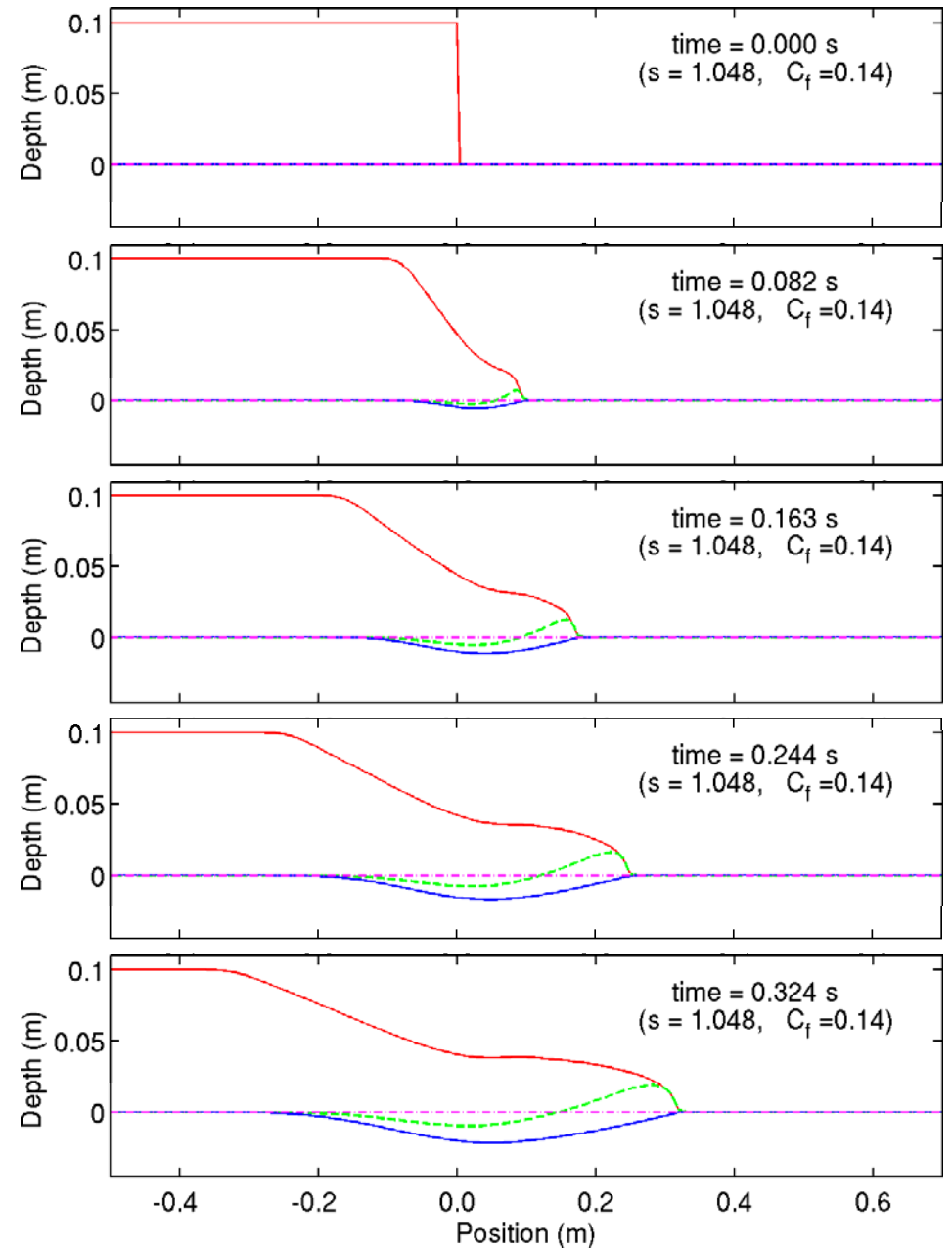
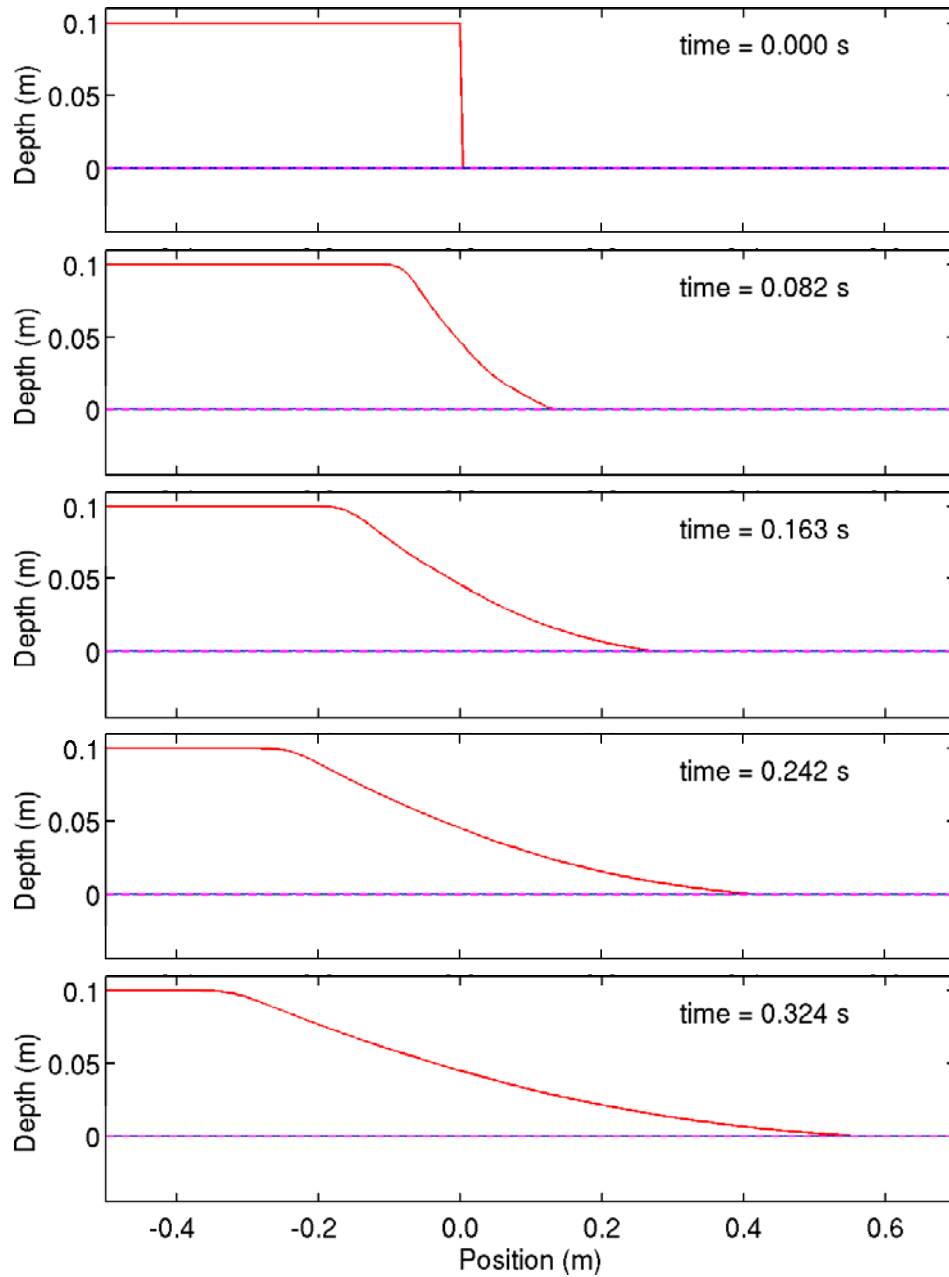
where jump conditions at bed

$$M_{\text{basal}} = \rho_G (\mathbf{u}_{\text{int}} - \mathbf{u}_G) \cdot \mathbf{n}_b = \rho_L (\mathbf{u}_{\text{int}} - \mathbf{u}_L) \cdot \mathbf{n}_b$$

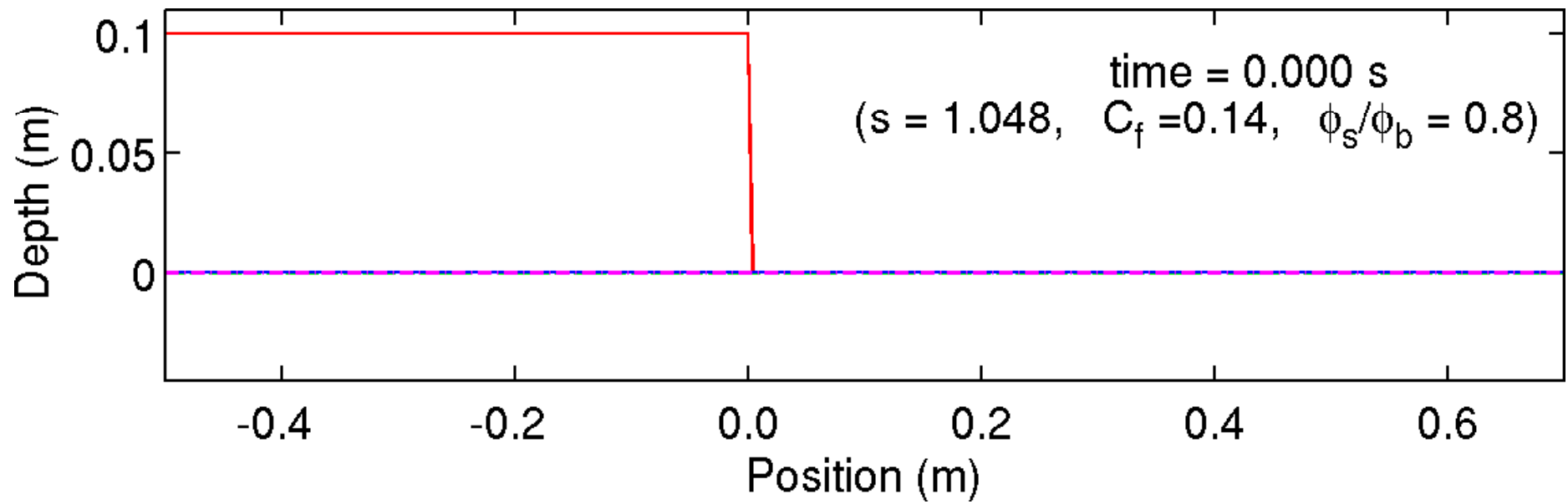
$$M_{\text{basal}} (\mathbf{u}_G - \mathbf{u}_L) = \mathbf{t}_G \mathbf{n}_b - \mathbf{t}_L \mathbf{n}_b$$

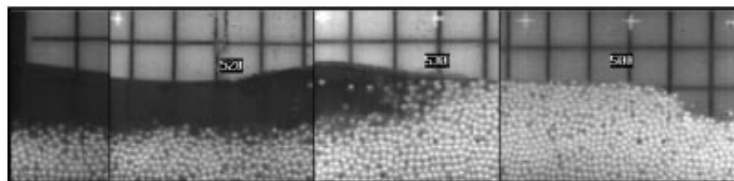
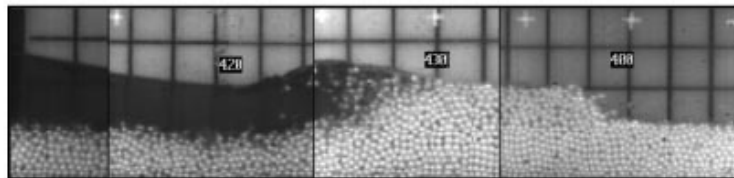
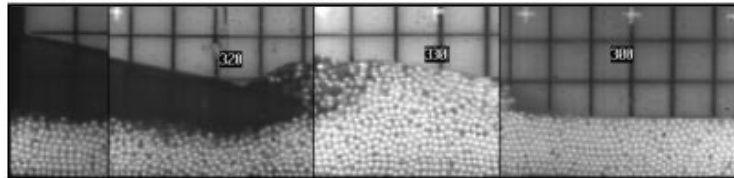
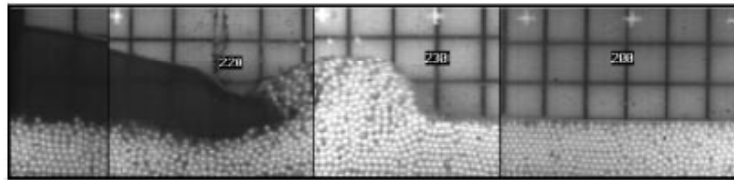
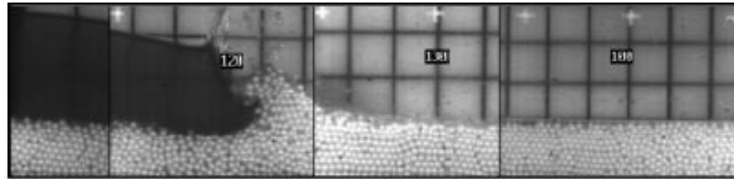
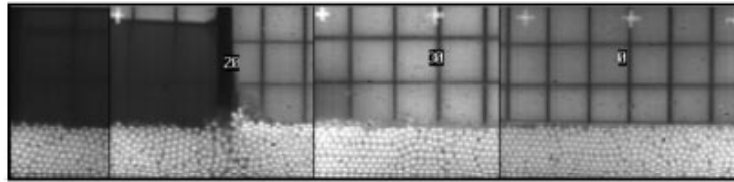


• **Effect of erosion**

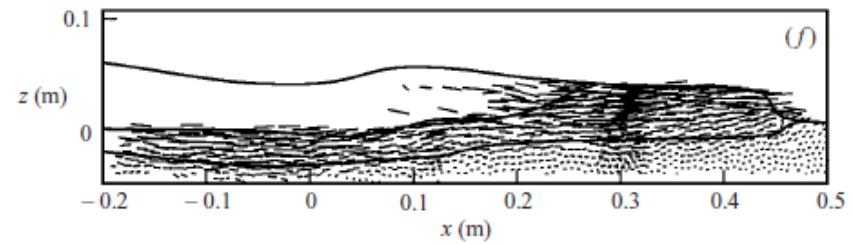
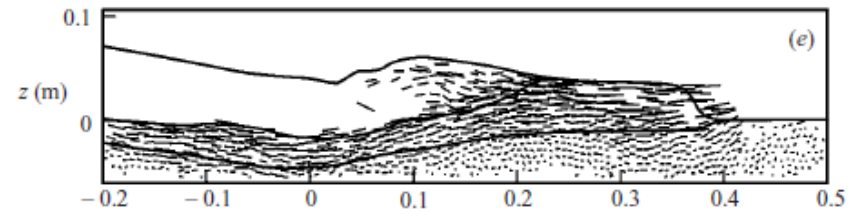
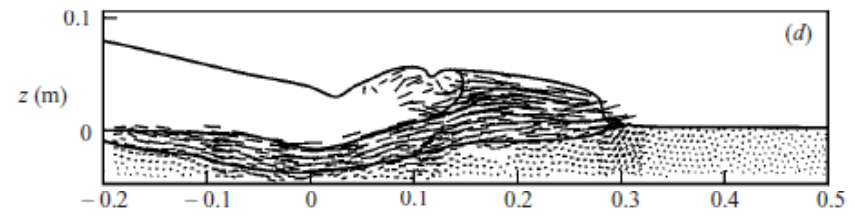
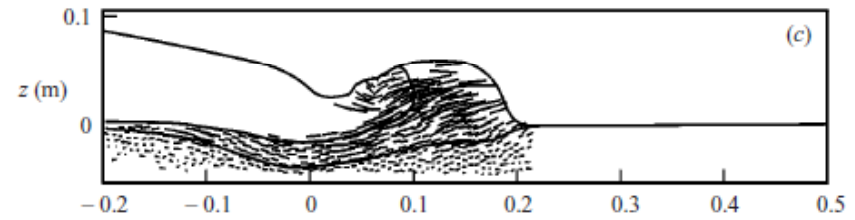
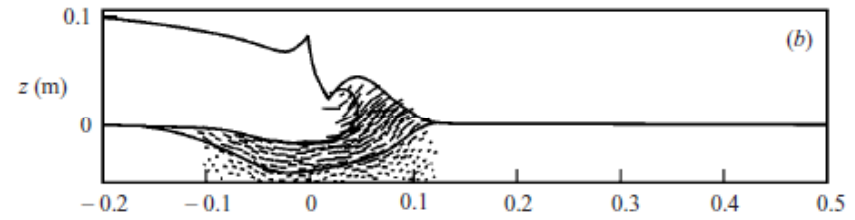


- *Result of simulation*

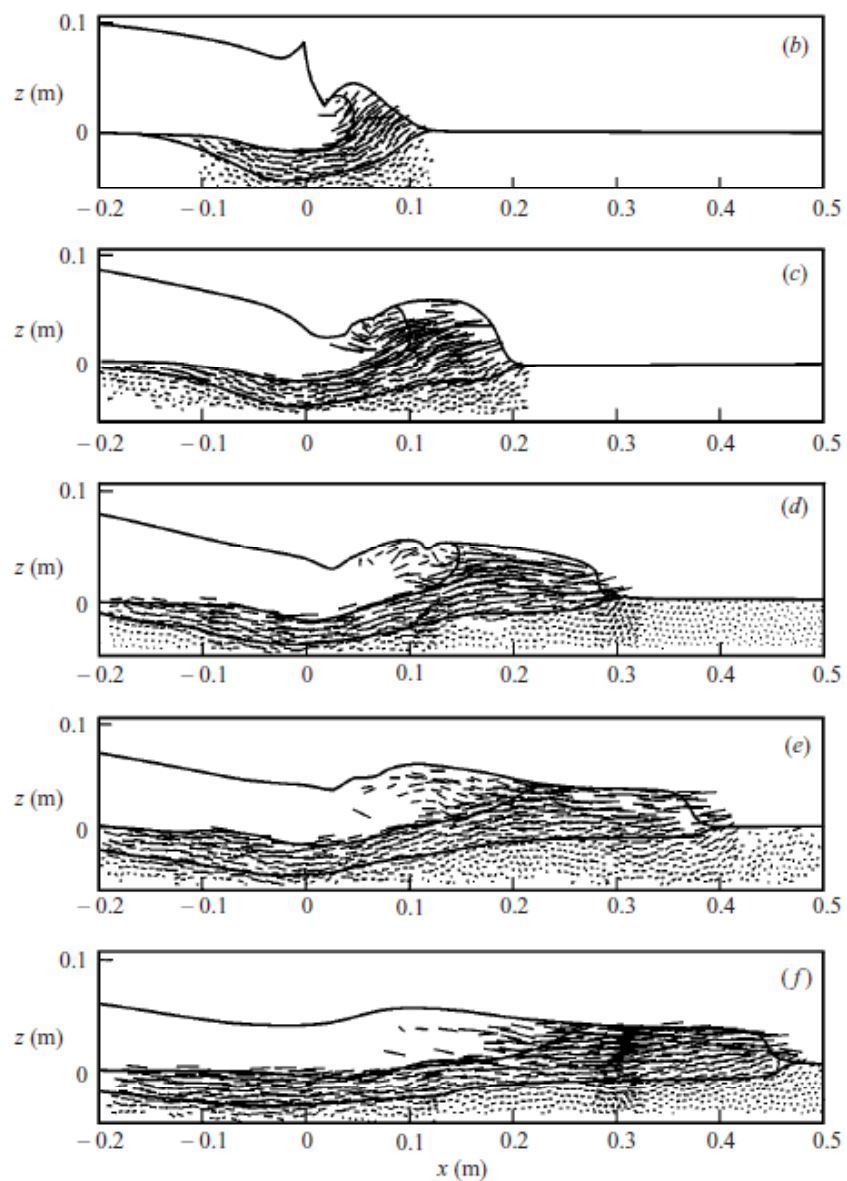




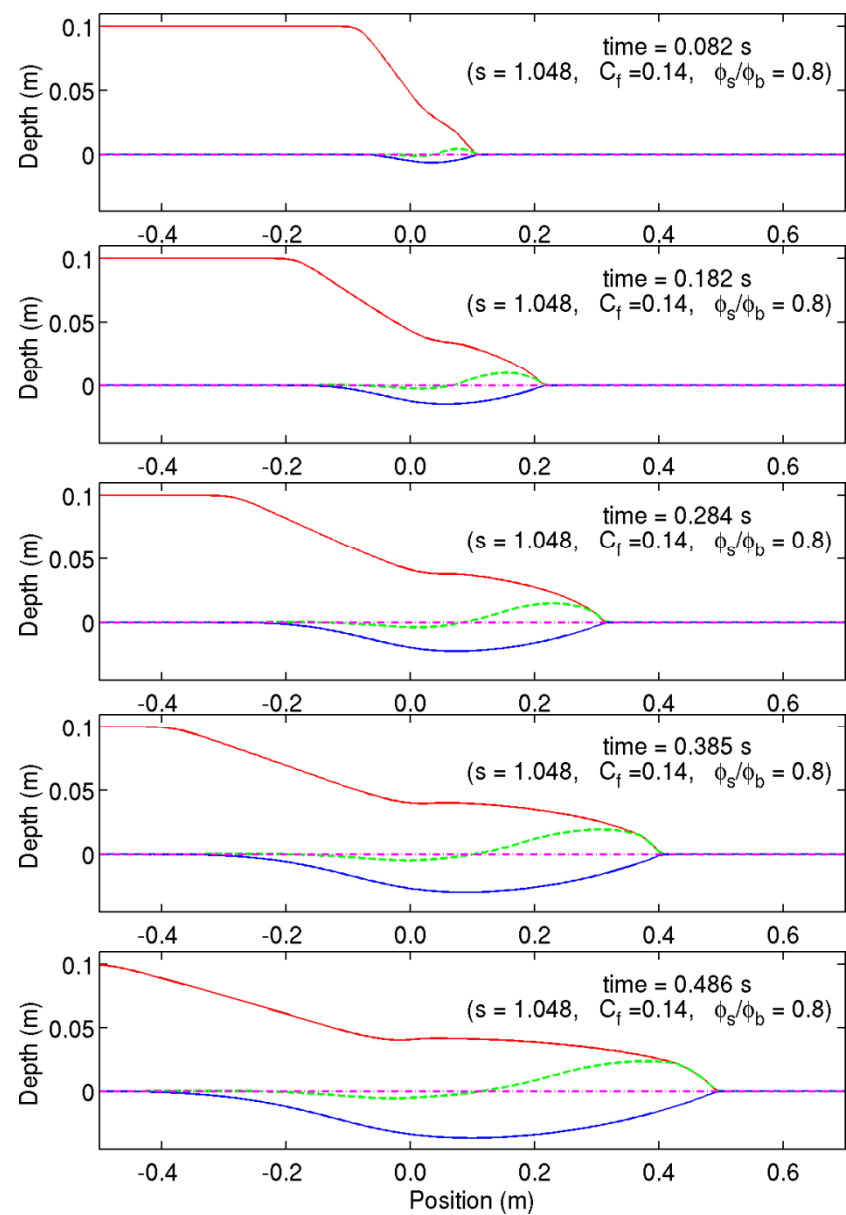
Experiment by Fraccarollo & Capart (2002)



Experiment by Fraccarollo & Capart (2002)



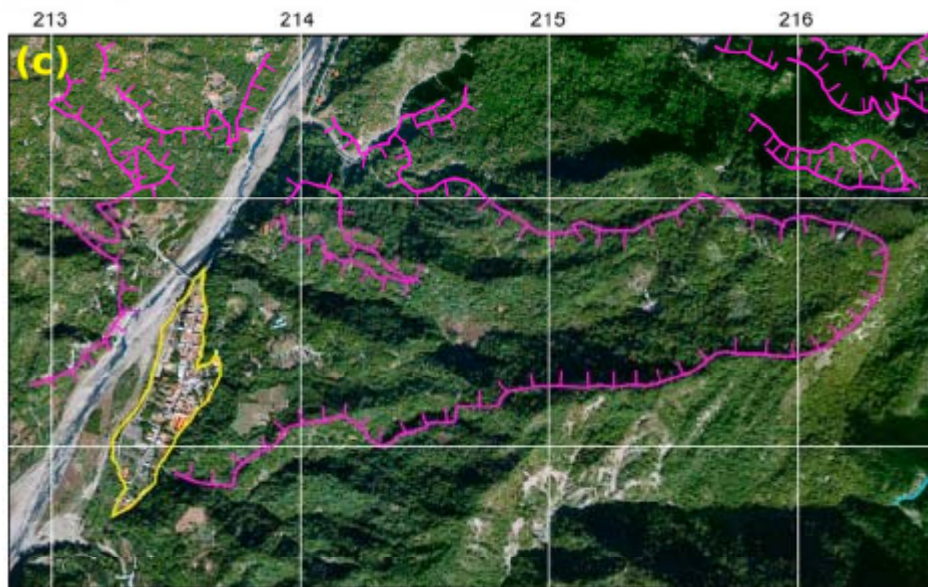
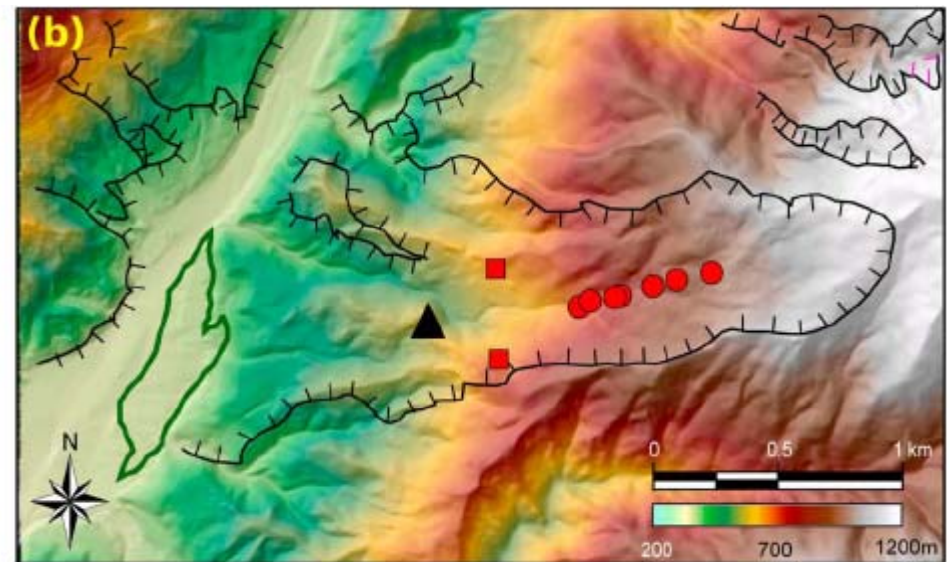
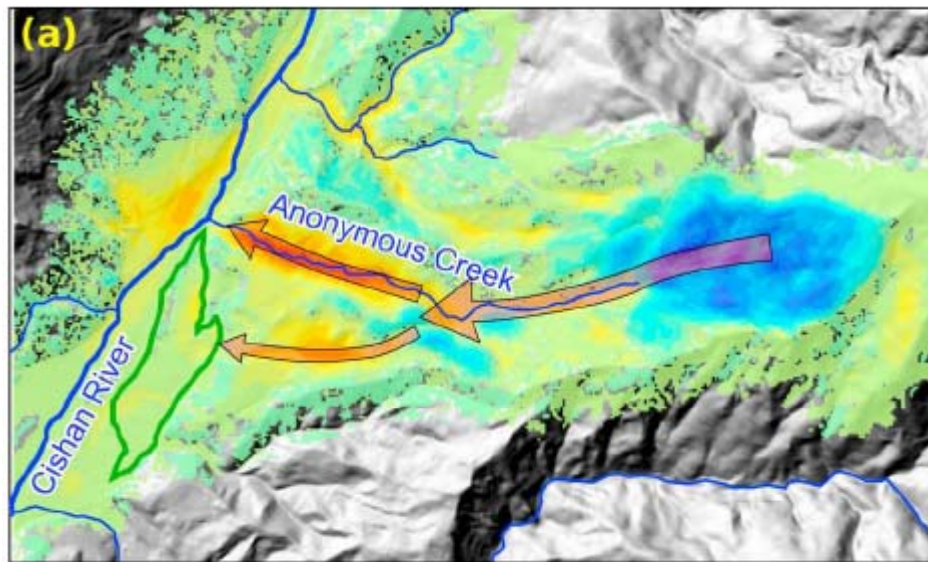
Numerical simulation by current method



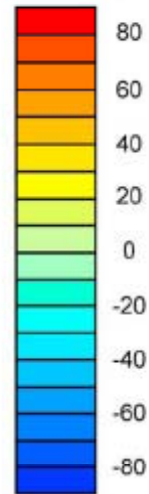
Reconstruction of the Hsiaolin Landslide Catastrophe, Taiwan 2009







by

*C. Y. Kuo, Y. C. Tai, C. C. Chen, K. J. Chang, A. Y. Siau, J. J. Dong,
R. H. Han, T. Shimamoto and C. T. Lee, JGR (2011)*



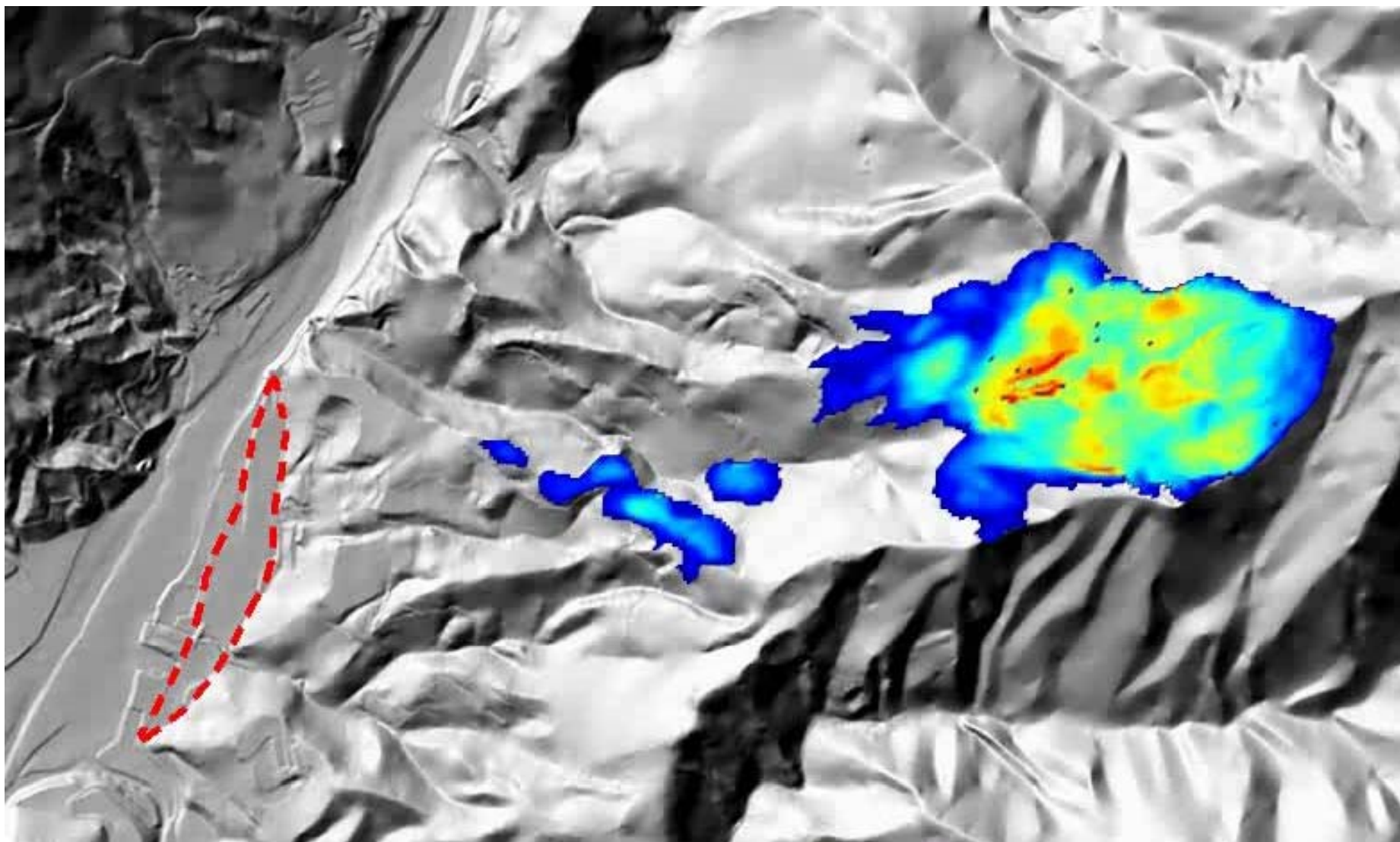
deposition
depth (m)



-  Shiaolin village
-  590 Height
- Sampling position**
-  Colluvium
-  Shale & Sandstone
-  landslide and deposition area
-  river



Animation



Optimal friction angle: 11.47°
(agree with the friction coefficient projection relation on the geometric constraint, and Staron & Lajeunesse ,2009)

Duration: 110 s

Volume: 26.17 Mm³

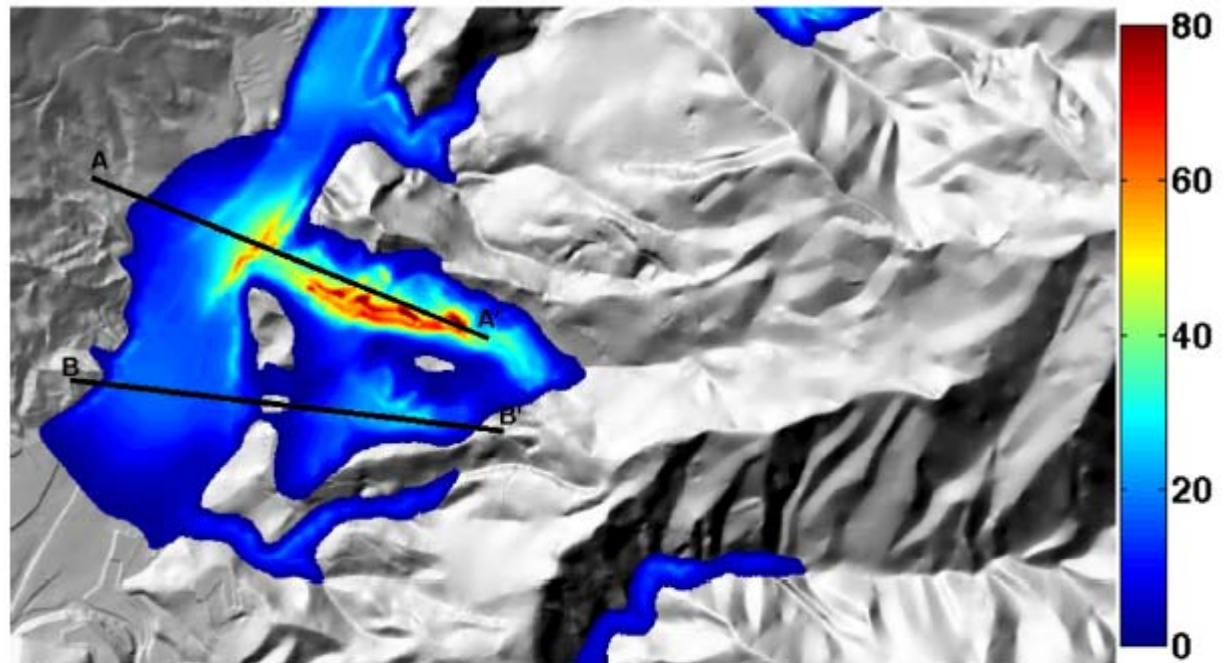
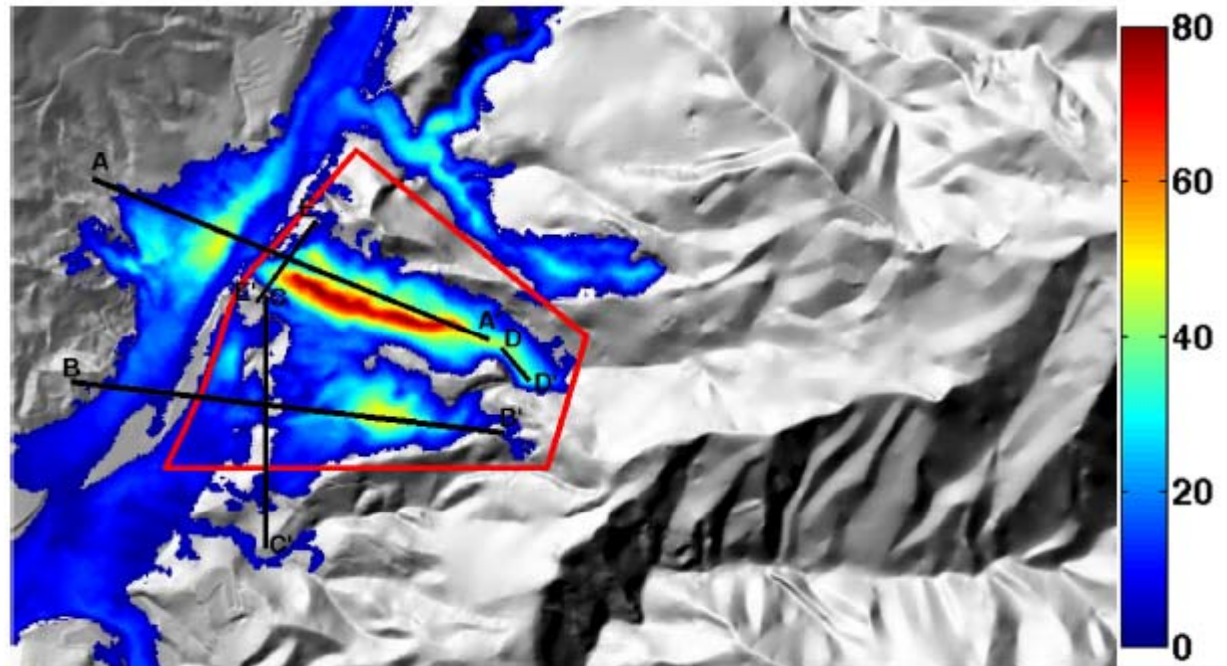
Comput. domain:

$3,710 \times 2,220 \text{ m}^2$

Mesh size: $10 \times 10 \text{ m}^2$

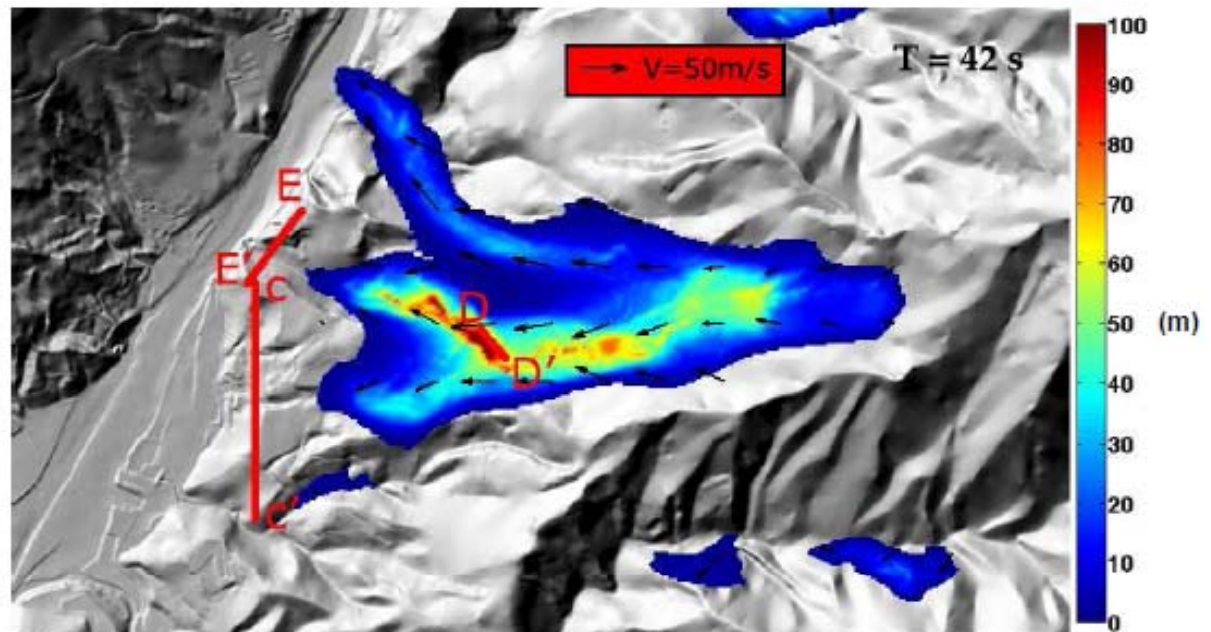
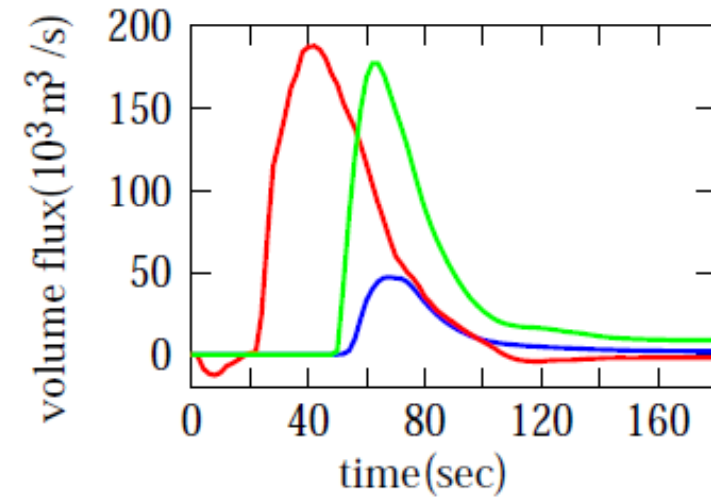
Optimization process:

$$h_{std}^2(\mu) = \min_{\mu} \frac{1}{A} \int_A (h(\mathbf{x}; \mu) - h_{meas}(\mathbf{x}))^2 dA$$



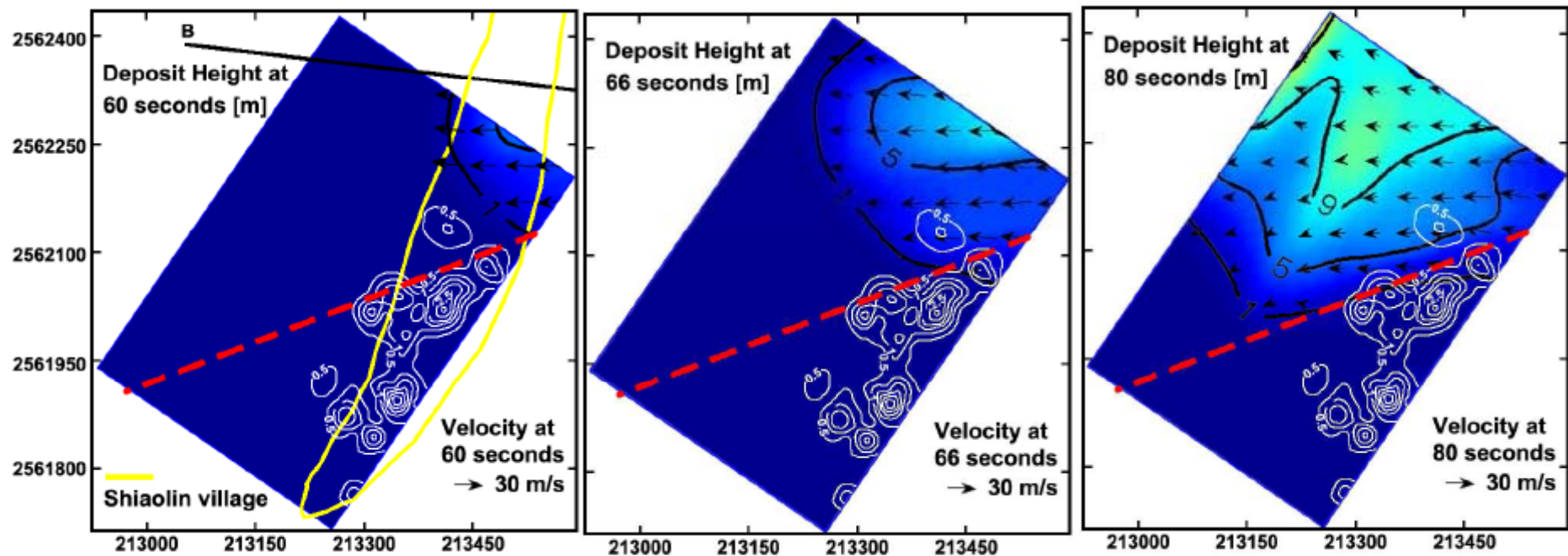
Transient volume fluxes

- The max. speed reaches about **50 m/s** in the anonymous creek valley.
- The flux across section **DD'** peaks at about **42 sec**.
- The two streams reach their maxima at about **62 and 64 sec**.
- About **7%** of the slid volume flow through the village.



Flow snapshots in comparison with the near-surface magnetic surveys

- Yellow line indicates the village area
- Red dashed line indicates the boundary of the magnetic anomalies (destruction front)
- Coincidence with the flow boundary (1 m depth)



Concluding remarks

Modeling part:

- *An alternative formulation for shallow hazardous flows over non-trivial and deforming topography is presented.*
- *The key features of this formulation are illustrated by the numerical examples.*

Application part:

- *Experimental validation: Upstream traveling wave; Chute experiment.*
- *Numerical example: Levee formation by Coulomb-mixture theory; Erosional dam-break shear flows.*
- *With the hydrodynamic model over general topography, the Hsiaolin landslide event can be favorably reconstructed.*

However,