Stability dependence of the relaxed eddy accumulation coefficient for various scalar quantities

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Received 20 March 2001; revised 23 September 2001; accepted 25 September 2001; published 27 April 2002.

The coefficient $b$ of the relaxed eddy accumulation (REA) technique was investigated by simulation studies using extensive high-resolution eddy correlation data sets of three different scalar quantities: the air temperature and the concentrations of CO$_2$ and H$_2$O. Measurements were performed in June–July 1995 over a senescent cereal field in the Rhineland-Palatinate region of southwestern Germany. The wide range of stability conditions during the experiment made it possible to describe the coefficient $b$ as a function of the dimensionless stability parameter $z/L$ within the framework of the Monin-Obukhov similarity theory. In good agreement with previous publications, a constant value of about 0.56 was obtained for unstable and near-neutral conditions if no dead band applied. For stable conditions, however, a significant increase with stability was observed which could be well described by a logarithmic functional relationship between $b$ and $z/L$. By additional dead-band simulation studies the relationship was extended to variable dead-band widths. The observed dependency on $z/L$ is interpreted in relation to the bivariate joint frequency distribution of the vertical wind speed and the scalar quantity.

INDEX TERMS: 0315 Atmospheric Composition and Structure: Biosphere/atmosphere interactions; 0394 Atmospheric Composition and Structure: Instruments and techniques; 3379 Meteorology and Atmospheric Dynamics: Turbulence; 0322 Atmospheric Composition and Structure: Constituent sources and sinks

1. Introduction

Several techniques have been developed for measuring trace gas exchange at natural surfaces, among which the eddy correlation (EC) technique is regarded as the most direct and reliable. Since turbulent transport occurs over a wide range of temporal and spatial scales down to very small and short-lived motions, the eddy correlation technique requires wind speed and trace gas measurements at a high temporal resolution and thus sensors with a response time faster than 1 s. Such instruments have been developed up to now only for a few compounds such as water vapor, ozone, carbon dioxide, methane, or nitrous oxide [Baldocchi et al., 1988]. For the majority of trace gases, however, sensors with a time resolution below 1 min are not available. To overcome this problem, Desjardins [1972, 1977] proposed and tested a modification of the eddy correlation technique called eddy accumulation. For this method, the required fast-response trace gas sensor can be replaced by fast-response sampling valves combined with slow analysis techniques. The so-called relaxed eddy accumulation (REA) represents a simplified version of the original approach. It was first suggested by Hicks and McMillen [1984] and formulated and discussed in detail by Businger and Oncley [1990]. Additional information on the theoretical background of REA and other eddy accumulation techniques is given, for example, by Wyngaard and Moeng [1992], Foken et al. [1995], Bowling et al. [1999], and Baker [2000].

With REA the turbulent trace gas flux is determined from the conditionally sampled mean updraft ($U$) and downdraft ($D$) concentrations. For the purpose of generality the following equation is formulated for the kinematic flux of an arbitrary scalar quantity $s$ that may represent a trace gas concentration or the air temperature. The overbar represents a Reynolds average and the prime an instantaneous deviation from this average.

$$ F_s = \overline{w's'} = b \sigma_w (\overline{s_U - s_D}). $$

The applicability of the REA approach depends on whether it is possible to understand and describe the quantitative behavior of the empirical coefficient $b$ as defined in (1). It is determined by the overall relationship between $w'$ and $s'$, which can be described by a bivariate joint frequency distribution (JFD). The basic theoretical model for describing a two-dimensional JFD is a joint Gaussian distribution that assumes a linear relationship between both quantities in combination with random noise. It is illustrated with the resulting conditional scalar means in Figure 1a. Wyngaard and Moeng [1992] show that the coefficient $b$ for such an ideal distribution is independent of the standard deviation and the correlation of $w'$ and $s'$ and thus equals a constant value:

$$ b = \frac{\sqrt{2\pi}}{4} \approx 0.627. $$

However, their large eddy simulations as well as several field experiments yielded values for $b$ which were, on average, somewhat lower than the theoretical value in (2) and partly showed considerable variability. Experimental values were mainly derived from high-resolution (eddy correlation) data sets of vertical wind speed and temperature, which is the easiest to measure of all scalar quantities. Average values for $b$ between 0.54 and 0.60 were found [Businger and Oncley, 1990; Baker et al., 1992; Pattey et al., 1993; Katul et al., 1996]. However, individual 30-min values sometimes showed a considerable scatter with ranges from 0.2 to 0.9 [Oncley et al., 1993; Beverland et al., 1996b].

It is still uncertain whether $b$ can be considered as constant or whether it varies systematically in relation to certain turbulence parameters as suggested by similarity theory. Andreas et al. [1998] proposed a numerical relationship between $b$ and the dimensionless

\[ \text{[Footnotes and references omitted for brevity.]} \]
2. Measurements and Methods

2.1. Site and Period

[7] Within the framework of the European Union (EU) program EXAMINE (Exchange of Atmospheric Ammonia With European Ecosystems), a field experiment was performed in June and July 1995 [Meixner et al., 1996; Neftel et al., 1998]. The experimental site (49°10' N, 8°16' E, 127 m above sea level (asl)) was located in the Rhine valley, southwestern Germany, about 2 km south of the village of Bellheim. The region is a relatively flat (level differences < ± 2.5 m within 2 km) and almost forest-free agricultural landscape. A survey of the vegetation around the measurement site during the field campaign is given in Figure 2. The field and cultivation pattern is typical for central Europe with many small and few bigger plots and a large variety of crop types. The figure also shows the position of the measurement towers at the northeastern corner of the main triticale field (canopy height ~1.35 m), which provided a fetch of about 300 m in the main southwest wind direction. In the east direction the fetch was clearly limited (about 100 m), whereas to the north, the adjacent sugar-beet field represented a strong influence for the flux measurements. This situation should allow to study the effect of various source area and fetch conditions on the flux measurement methods.

2.2. Instrumentation

[8] Eddy correlation measurements were performed for the trace gas fluxes of CO$_2$ and H$_2$O as well as for the fluxes of sensible heat and momentum. The measurement system consisted of a three-dimensional sonic anemometer (Gill Instruments, Solent Research 1012 R2) for the detection of the wind vector and air temperature and a closed-path CO$_2$/H$_2$O gas analyzer (LI-COR, Li-6262). The sonic anemometer and the inlet for the trace gas analyzer were installed at a height of 4.25 m above ground (hence about 3.35 m above the zero-plane displacement level) on a light mobile tripod tower. The 1/4 inch inlet tube for the CO$_2$/H$_2$O gas analyzer was fixed directly to a supporting arm of the sonic anemometer (12 cm from the center of the sensor paths) without causing additional disturbance to the wind field. The air was pulled through Teflon tubing of 6 m in length down to the analyzing unit at the ground. The sample airflow was maintained by a rotation pump (Brey, G12/07-N) positioned downstream of the analyzer and regulated by the output of a mass flowmeter (MKS, type 358C) to maintain a constant flow rate of 7.2 sL min$^{-1}$ (1sL = 1 liter at standard temperature and pressure conditions). This setup created a turbulent flow regime in the sampling tube with Reynolds numbers above 2500, an effective response time of about 0.1 s, and a delay time of about 1 s.

[9] The entire high-resolution time series of the eddy correlation instruments were recorded and stored in their raw state by a Notebook computer with an 80386 processor. The sonic anemometer recorded and transferred their data at a rate of 20.83 Hz (1250 samples per minute); the analog signals of the trace gas sensors, however, were digitalized at a rate of only 10 Hz.

2.3. Data Evaluation and Rejection

[10] Values of the REA coefficient $b$ were simulated numerically from the fast-response eddy correlation time series. They were calculated for the fluxes of the various scalars $s$ (i.e., temperature, CO$_2$, and H$_2$O) by rearranging (1):

$$b_s = \frac{\bar{w}'s'}{\sigma_u (\bar{s}_u - \bar{s}_D)}.$$  

[11] All averages (denoted by the overbars) were calculated for 10-min intervals. This is long enough to cover the relevant frequency range of the turbulent fluctuations at the chosen measurement height [cf. Kaimal and Finnigan, 1994]. On the other hand, it is short enough for an effective removal of nonturbulent trends. For minimizing possible nonideal effects in the eddy correlation measurements, a thorough preconditioning of the data sets was performed. It included the common correction procedures.
usually applied for the eddy correlation method, i.e., wind vector coordinate rotation, delay time adjustment, and linear detrending. Corrections for correlated density effects [Webb et al., 1980] were not applied to the time series. They affect the conditional mean difference (\( \bar{s}_T - \bar{s}_P \)) and the eddy covariance flux in the same way [Pattey et al., 1992], and thus the ratio of both quantities in (3) is not influenced. The effect of high-frequency attenuation (mainly due to sensor separation) could be corrected in the eddy correlation fluxes by a spectral transfer model as proposed, for example, by Moore [1986]. A correction of this effect in the original time series and thus in the simulated conditional scalar means is not possible. However, it was found by simulation experiments on the high-resolution temperature time series that the attenuation effect on the conditional means is very small [Ammann, 1999]. Additionally, cases with an estimated flux attenuation (and subsequent correction) of more than 10% were excluded from the evaluation.

[12] To avoid unfavorable environmental conditions, some additional rejection criteria were applied to the REA simulations, as described in the following. Beside instrumental problems, such as logging failures or water vapor condensation in the sampling tube, there were also methodological limitations for the flux determination. The most important one is the requirement for fully developed turbulence and the applicability of the surface layer similarity. The determining parameter is the atmospheric stability \( z/L \). The limitation in the unstable range was not important in the present case, because no extremely unstable conditions occurred. In the stable range, the present evaluation was limited to \( z/L < 2 \), leading to a rejection percentage of 13.3%. An additional indicator for the existence of fully developed undistorted turbulence is the correlation coefficient between the horizontal and the vertical wind component \( R_{wv} \). Kaimal and Finnigan [1994] give an average \( R_{wv} \) value of \( -0.35 \); data were rejected if \( R_{wv} \geq -0.1 \). Positive or near-zero values (rejection percentage 19.5%) might have been caused by flow distortion (sensor head, tower assembly, or nearby obstacles) or nonstationarity effects. They mainly occurred under low-wind conditions.

[13] An additional threshold for rejecting small fluxes was supposed to prevent large relative uncertainties in the \( b \) values. The threshold limits for the different scalar fluxes are listed in Table 1. They were chosen as about 5–10% of the typical range of observations. The fetch condition (depending on wind direction) was not used here as a rejection criterion, because the effect of inhomogeneous source areas on the REA method is not well known yet and will be analyzed in the following.

### 3. Results

#### 3.1. Dependence on Stability

[14] The first part of the evaluation focuses on the \( b \) values determined by (3) without any dead-band application. These are denoted in the following as \( b_0 \). According to Obukhov [1971], dimensionless turbulence characteristics in the surface layer, like the REA coefficient \( b \), should either be constant or be dependent only on a dimensionless stability parameter like \( z/L \). Thus for an overview of the obtained \( b_0 \) values for the three different scalar quantities, the results are displayed as box plots in Figure 3 for eight classes of the stability \( z/L \). The median and percentile statistics used in the box plot minimize the influence of extreme individual values (outliers) and are thus generally more meaningful than the corresponding arithmetic means and standard deviations. The scatter of the individual values as indicated by the interquartile range (between lower and upper box boundaries) is typically between 5% and 20%. It is lowest for temperature and highest for \( CO_2 \). For water vapor, too few values for box-plot statistics were available in the stable range owing to the rejection criteria applied. The main reasons were nighttime fluxes close to zero and frequent occurrence of condensation water in the sampling line.

[15] In the unstable range the median \( b_0 \) values show no systematic dependence on \( z/L \). The variability among the four stability classes as well as the difference between the investigated scalars is very low (in the order of 2%) and therefore not significant in the context of micrometeorological similarity analysis. Thus all median values were averaged to a total mean \( b_0 \) of \( 0.56 \pm 0.01 \), which is considered to be representative for the entire unstable range. The value is plotted as a dashed-dotted line in Figure 3. Under stable conditions, however, the values show a moderate but systematic increase with stability up to 0.63 and higher. To describe the behavior of the \( b_0 \) under stable conditions analytically, a regression function had to be fitted to the measured data. It has to be considered that it is difficult to choose an appropriate functional form if the scatter of the data points is large. Moreover, the common least squares regression algorithm is particularly sensitive to non-Gaussian characteristics of the sample points, i.e., to outliers and asymmetric distributions. This problem seemed to be least pronounced for the \( b_0 \) values for temperature (compare Figure 3), which show a good agreement of the arithmetic averages and the median values in the individual classes. In this case, a simple linear function according to (4).
regression could be applied, which resulted in the numerical relationship

$$b_0 = 0.63 + 0.035 \log_{10}(z/L).$$

[16] The regression line is plotted in Figure 3 together with the box plots for the different stability classes. For CO$_2$, the observed distributions are much more unfavorable for the application of a least squares regression. There is a considerable difference between arithmetic averages and medians in stable conditions. An asymmetric distribution with large deviations on the upward side leads to arithmetic means that are generally higher than the more representative medians. In this case, least squares methods failed to give a useful functional estimate for the relationship of $b_0$ and $z/L$. However, the relationship (4) derived for temperature also seems to fit the median $b_0$ values for CO$_2$ fairly well. Only in the most stable class, a slight deviation of the median, and also a significantly larger scatter, is observed. The nocturnal CO$_2$ flux turned out to be particularly suitable for the study of stable conditions, because it was of similar magnitude as the corresponding daytime values. Therefore it was much less affected by measurement errors than the very small nocturnal H$_2$O flux.

### 3.2. Dependence on Other Parameters

[17] As mentioned above, $z/L$ is the most obvious influencing parameter for the coefficient $b$ according to similarity theory. The identification of other potential controlling variables is generally problematic, because most of them are somehow correlated to the stability through characteristic variations in the diurnal cycle. It is thus difficult to decide whether an observed correlation is due to a direct physical influence or due to indirect dependency of both quantities on $z/L$. This problem can be avoided, if only unstable cases are considered, where no systematic relation between $b_0$ and $z/L$ is found. In Figure 4, values of $b_0$ for unstable conditions are plotted against the wind direction corresponding to different fetch length (compare Figure 2). The data points crowd within the SW wind sector (representing generally a large fetch) and in the NW sector (representing small fetch lengths). However, no significant dependence of the $b_0$ values can be observed.

[18] To check the existence of other unknown controlling parameters, the individual $b_0$ values for different scalars are plotted against each other (Figure 5). Again, the data are confined to unstable conditions to exclude the stability effect. The resulting correlation coefficients are very small and hardly significant. This indicates that $z/L$ is the only important influencing parameter for the coefficient $b_0$ and that the residual variability has the characteristic of a random noise. The difference of the scatter for temperature and trace gases can be attributed mainly to the lower noise level of the temperature measurements and the exclusion of sensor separation effects.

### 3.3. Effect of a Dead Band

[19] The application of a dead band in the REA method corresponds to a simple binary weighting of the air sampling with the vertical wind speed. As illustrated in Figure 1b, the conditional scalar means get generally larger with a growing dead-band width $w_d$ for an ideal Gaussian distribution. The exact quantitative effect could not be determined analytically, like the zero-dead-band value in (2). Therefore the Gaussian distribution was integrated numerically for variable $w_d$. The resulting $b$ values were found to be described very well by a function of the form

$$b = b_\infty + (b_0 - b_\infty) \exp \left( -\frac{w_d}{\sigma_w} \right), \quad 0 \leq w_d/\sigma_w \leq 2. \quad (5)$$

[20] For the purpose of general applicability, the dead-band half-width $w_d$ was normalized with the standard deviation of the vertical wind speed $\sigma_w$. Function (5) describes an exponential decay with the initial zero-dead-band value $b_0$ as given in (2) and an asymptotic baseline value $b_\infty$ for an infinite dead-band width. The numerical results for the fitting parameters are given in Table 2.

[21] To investigate the dead-band effect for real environmental conditions, simulation studies with variable dead bands were performed on the experimental eddy correlation data sets. Figure 6 shows the simulated $b$ values for temperature as a function of the normalized dead band. Stable and unstable cases are separated in two individual plots. Again, the data were divided into eight stability classes as described in Figure 3, and functions similar to (5) were fitted to the data by adjusting the parameters $a$ and $b_\infty$. The $b_0$ values were not fitted in this regression procedure but prescribed from the zero-dead-band analysis, as presented in section 3.1. Within the unstable range, no significant differences were found among the classes, and therefore only one overall function was determined. It is plotted as a line in Figure 6a, and the corresponding parameter values are given in Table 2. In the stable range, a relatively constant value was found for the exponential coefficient $a$, but a systematic variation with $z/L$ was found for the baseline parameter $b_\infty$. Similar as for $b_0$, it could be described by a linear relationship with log$_{10}(z/L)$. The corresponding numerical values are listed in Table 2. To illustrate the shape of the fitted functions, two curves are plotted in Figure 6b, one for near-neutral and one for extremely stable conditions. The difference between the curves is largest for zero dead band and reverses its sign at about $w_d/\sigma_w = 0.65$.

[22] The dead-band evaluation for H$_2$O and CO$_2$ is not shown here. The corresponding results have a bigger scatter than the

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**Figure 4.** REA coefficient $b_0$ for temperature plotted versus wind direction for unstable conditions.

**Figure 5.** Comparison of individual $b_0$ values for different scalar quantities under unstable conditions: (a) CO$_2$ versus $T$; (b) CO$_2$ versus H$_2$O.
results presented for temperature, which increased the uncertainty of the regression procedure. However, no significant differences among the three scalars were found.

4. Discussion

4.1. Findings for Zero Dead Band

[23] Most studies published thus far only give an overall average value for the REA coefficient $b_0$ with no dead band applied. This is either because the authors found no significant dependence on environmental parameters or because they did not specifically investigate this problem (partly because of the limited data sets available). The average $b_0$ values reported for unstable and near-neutral daytime conditions range within 0.56–0.58 [Pattey et al., 1993, 1995; Gao, 1995; Katul et al., 1996; Beverland et al., 1996a] and are thus comparable to the constant value of about 0.56 found in this study (compare Figure 7). The literature results were obtained from measurements over a variety of different surface/vegetation types, including bare soil, peatland, grassland, soybeans, corn, and deciduous forest. Only Andreas et al. [1998] suggest a nonconstant relationship between $b_0$ and $z/L$ in the unstable range. Their functional relationship is also plotted in Figure 7 in comparison to the present results. It indicates $b$ values around 0.55 for extremely unstable conditions and decreasing values down to about 0.52 for near-neutral situations. Especially the latter value is significantly lower than our findings and most other published results. Andreas et al. [1998] argue that most other authors averaged their results over the entire stability range, which clearly does not hold for the present study.

[24] Only very few observations of $b_0$ under stable conditions are reported in the literature. Most of them were not treated separately but incorporated into the overall mean [e.g., Baker et al., 1992]. The evaluations of the present data set indicate a systematic increase of the coefficient $b_0$ with $z/L$ for stable conditions described by (4). This finding agrees at least qualitatively with the individual data of Businger and Oncley [1990] and the functional relationship derived by Andreas et al. [1998], as displayed in Figure 7. The difference to the latter relationship is mainly in the near-neutral to moderately stable range. There, the function of Andreas et al. gives lower values mainly because it is adjusted to the low near-neutral values on the unstable side (see above).

[25] The present findings disagree significantly with the results of Pattey et al. [1995] obtained over bare soil. They report a distinctly lower average $b_0$ value of 0.49 for stable nighttime conditions as compared to daytime, yet with a large variability of ±0.13. They explain their finding with large concentration changes at low frequencies and generally small fluxes. Such effects were minimized in the present evaluation by the use of restrictive rejection criteria (for small fluxes and for extreme stability conditions $|z/L| > 2$) and by an effective detrending on a 10-min basis.

[26] From the similarity hypothesis for scalar transport it can be assumed that the coefficient $b$ for trace gases is identical to that for

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### Table 2. Numerical Parameters of the Fitting Function (5) Describing the Dead-Band Effect on the Ideal Gaussian JFD and on the Temperature EC Data Set Depending on the Stability Parameter $z/L$

<table>
<thead>
<tr>
<th>$z/L$</th>
<th>$b_0$(Fixed)</th>
<th>$b_1$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal joint Gaussian distribution</td>
<td>0.627</td>
<td>0.144</td>
<td>1.04</td>
</tr>
<tr>
<td>Experimental EC data: $z/L = -2 \ldots 0$</td>
<td>0.56</td>
<td>0.28</td>
<td>1.71</td>
</tr>
<tr>
<td>$z/L = +0.01 \ldots +2$</td>
<td>function (4)</td>
<td>$0.18 - 0.027 \log_{10}(z/L)$</td>
<td>1.25</td>
</tr>
</tbody>
</table>

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![Figure 6](image-url). The $b$ values derived from EC time series of temperature with variable dead-band width and fitted regression curves according to (5) and Table 2: (a) unstable conditions, (b) stable conditions. The regression curves are for near-neutral conditions with $0 < z/L < 0.06$ (dashed) and extremely stable conditions with $0.6 < z/L < 2$ (solid).
temperature (sensible heat). Only few studies have investigated this question up to now [Pattey et al., 1993; Katul et al., 1996]. They found a relatively good agreement for temperature, H$_2$O, and CO$_2$ under unstable conditions. The results of the present investigation on $T$, H$_2$O, and CO$_2$ data support the general similarity of $b_0$ for all scalar quantities. However, it has to be noted that the good agreement of $b_0$ values for temperature and CO$_2$ under stable conditions only applied to the median statistics and not to arithmetic averages (compare Figure 3), which were strongly affected by the non-Gaussian distribution of individual values. The influence of instrumental limitations (e.g., high-frequency damping) on the estimated $b_0$ values is highly unlikely, because they would have mainly affected the CO$_2$ but not the temperature measurement. A systematic effect of limited fetch can be excluded for unstable and neutral conditions. Yet it is very difficult to identify in the stable range and thus cannot be generally excluded.

[27] From the findings and considerations presented above, the following overall similarity relationship between the REA coefficient $b_0$ and the stability $z/L$ is proposed for all scalar quantities:

$$\begin{align*}
  b_0 &= \begin{cases} 
    0.56 + 0.035 \log_{10}(z/L) & \quad -2 \leq z/L \leq +0.01 \\
    0.63 & \quad +0.01 \leq z/L \leq +2.
  \end{cases}
\end{align*}$$

(6)

[28] To check the general validity of the unique relationship between $b_0$ and $z/L$, it was used for the flux evaluation of all scalar quantities ($T$, H$_2$O, CO$_2$) according to (1). The simulated REA fluxes were then compared to the original eddy correlation fluxes.

Figure 8a shows the results for CO$_2$. The agreement is excellent with correlation coefficients close to 1.

[29] It was suggested by some authors [Oncley et al., 1993; Pattey et al., 1993] not to use a general prescribed $b$ for trace gas flux measurements but to adopt the individual $b$ values derived from simultaneous EC measurements of temperature. This approach was evaluated too, and the resulting CO$_2$ fluxes are validated in Figure 8b. The correlation is high as well but significantly lower than in Figure 8a. As already shown in Figure 5, the deviations of individual $b_0$ values from relationship (6) are uncorrelated for the different scalars and show a purely random characteristic. Thus the lower correlation in Figure 8b results from the combined random errors in both the temperature and the trace gas measurements. It can be concluded that there is no undetected systematic influence on $b_0$ beside the described dependence on $z/L$.

[30] The observed random variability for unstable conditions is more or less in agreement with (or even smaller than) the general uncertainty of finite time turbulence measurements discussed, for example, by Lenschow et al. [1994]. For stable conditions the scatter (especially for CO$_2$) is larger than expected, probably because of nonstationarity effects in combination with small fluxes.

4.2. Findings for Dead-Band Application

[31] In section 3.3 the effective $b$ values resulting from dead-band applications could be adequately described by functions of the general form given by (5). The effect of stability resulted in slightly varying numerical parameters listed in Table 2. The corresponding curves are displayed in Figure 9 in a combined form as a shaded area. It is limited by the extreme stable and unstable cases. For comparison the function derived from the ideal joint Gaussian distribution is also plotted; it is very close to the empirical relationship for stable conditions. Similar evaluations had already been performed by Businger and Oncley [1990] and Pattey et al. [1993], however, on the basis of limited data sets. The first study examined only 25 individual cases with near-neutral and unstable conditions, whereas the second study was confined to unstable conditions only. Businger and Oncley fitted a simplified form of (5) to their data with $b_\infty = 0$ and $\alpha = 0.75$, while Pattey et al. presented a more complex functional relationship, which, however, also can be transformed into the form of (5) with $b_0 = 0.57$, $b_\infty = 0.32$, and $\alpha = 1.96$. Both functions are displayed in Figure 9. The result of Pattey et al. agrees fairly well with the unstable limit of the present study, whereas the function proposed by Businger and Oncley follows more the stable limit. Both models are confined to smaller dead-band widths ($0 < w_d/\sigma_w < 0.8;0.9$).
All findings are fairly consistent and support the systematic dependence on stability observed in the present study. As for the zero-dead-band value discussed in section 4.2, no systematic difference in the dead-band function for various scalar quantities could be found [see also Pattey et al., 1993]. An intriguing feature represented in Figure 9 is the minimum variability of $b$ at a normalized dead-band width of about 0.7 ($b_C = 0.37$). It is caused by the crossing of the functions for different stabilities. A similar behavior was already observed by Wesely and Hart [1994] for seven exemplary half-hour runs. On the basis of their observations, they recommended an online adjustment of the normalized dead band to the specific value of 0.75.

4.3. Deviations From the Ideal Joint Gaussian Frequency Distribution

It was mentioned above that an ideal joint Gaussian distribution between the vertical wind speed and the transported scalar quantity yields a well-defined $b$ value, which is even constant ($b_0 = 0.627$) for a zero dead band. Consequently, the observed systematic deviation from the ideal Gaussian $b$ value, especially in unstable conditions, has to be connected to a departure of the actual JFD from the Gaussian form. Figure 10 shows experimental JFDs of two typical measurement intervals as contour plots: Figure 10a for an unstable (daytime) and Figure 10b for a stable (nighttime) case. The displayed distributions for temperature can be regarded as fairly representative for all scalar quantities investigated (for both flux directions), since no significant differences were found among them [cf. Ammann, 1999].

The deviation from the ideal joint Gaussian distribution (Figure 1a) is most obvious for the unstable case (Figure 10a), which is highly skewed with a peak frequency away from the center. This peak can be assumed to represent the almost constant scalar profile of the convective boundary layer connected to relatively slow downdraft motions [Wyngaard and Moeng, 1992]. The scalar deviations in the opposite direction are obviously larger but occur less frequently. They are connected to upward motion and thus represent near-surface conditions below the measurement height. The stable case (Figure 10b), in contrast, seems to follow the ideal joint Gaussian distribution reasonably well; it is fairly symmetric in both dimensions, and the highest frequency is near the center of the coordinate system. However, the deviation from the ideal JFD is still sufficient to cause a significant reduction in the observed $b_0$ values compared to the ideal Gaussian value of 0.627 (about 0.60 at $z/L = +0.1$). Thus an objective method is necessary to quantify the type and magnitude of deviation in the JFD.

Figure 10. Normalized measured JFD of vertical wind speed $w$ and temperature $T$ displayed as contour plots (isoline interval: 0.015): (a) 4 July 1995, 1650–1700 LT (local time: MEST), unstable conditions with $z/L \approx -0.2$, and (b) 15 July 1995, 0010–0020 LT, stable conditions with $z/L \approx +0.1$.

Figure 11. (a) Experimental REA $b_0$ values derived from the one-dimensional $w$ distribution alone according to (9), dependent on stability; the horizontal line indicates the ideal Gaussian value 0.627. (b) Deviation from linearity of the $w$-$T$ distribution determined as the ratio of the ideal linear slope (8) and the true slope (7) of the linear regression line.
According to Baker et al. [1992] the joint Gaussian distribution implies two principle characteristics: (1) a Gaussian form of the one-dimensional distributions of \( w' \) and \( s' \) and (2) a linear relationship between the two quantities. The second requirement signifies that the data points are randomly distributed around a linear regression line \( s' = \alpha + \beta w' \). The offset \( \alpha \) is equal to zero due to the use of the prime quantities, and the slope \( \beta \) is defined as [Sachs, 1982]

\[
\beta \equiv \frac{w's'}{\sigma_w \sigma_s} = R_{ws} \frac{\sigma_s}{\sigma_w}, \tag{7}
\]

According to Sachs [1982] and Baker et al. [1992] the linear relationship between two quantities implies that if the data set is divided into subgroups (here updrafts with \( w > 0 \) and downdrafts with \( w < 0 \)), the means in each group \((\bar{w}_U, \bar{s}_U)\) and \((\bar{w}_D, \bar{s}_D)\) fall on the regression line. Consequently, the slope \( \beta \) can be estimated from the averages of the subgroups in the following way:

\[
\beta_{\text{linear}} = \frac{\bar{s}_U - \bar{s}_D}{\bar{w}_U - \bar{w}_D}. \tag{8}
\]

If the assumption of linearity is valid, the combination of (7) and (8) with (1) yields the following simple expression for the REA coefficient depending solely on the \( w \) distribution:

\[
b_{0,w} = \frac{\sigma_w}{\bar{w}_U - \bar{w}_D}. \tag{9}
\]

As pointed out by Katul et al. [1996], this expression is directly related to the flatness or kurtosis (tendency for long tails) of the \( w \) distribution. Thus the skewness of the single \( w \) and \( s \) distributions have no effect on \( b \) so long as they are similar (i.e., linearly related) to each other. For an ideal Gaussian distribution, \( (9) \) is equal to the ideal value 0.627, whereas longer tails lead to higher values because they have a stronger influence on \( \sigma_s \) than on the conditional averages. The corresponding experimental results are displayed in Figure 11a. They indicate a generally positive deviation of \( b_{0,w} \), from the Gaussian value, whereas the “true” \( b_0 \) values determined according to (3) showed a predominantly negative deviation (compare Figure 3). It thus can be concluded that the assumption of linearity used for the derivation of (9) is violated.

The deviation of the observed JFDs from the ideal linear model may be quantified by the ratio of the ideal linear and the true regression line slope defined in (8) and (7), respectively. The reference value for the ideal case is \( \beta_{\text{linear}}/\beta \approx 1 \). The corresponding results are plotted in Figure 11b against stability. They show relatively constant low values in the unstable range and increase toward unity in the stable range. This systematic deviation is more or less equivalent to the behavior of the experimental \( b_{0,w} \) values in Figure 3. Hence a generally strong relationship between \( b_0 \) and the nonlinearity of the joint frequency distribution can be stated. Similar results were already observed by Baker et al. [1992] and Katul et al. [1996], yet only for small data sets limited to unstable daytime cases. The present results confirm and complete their findings for a broad range of stability conditions.

According to Milne et al. [1999] the nonlinearity of JFDs may also be described by more complex mathematical models such as the bivariate Gram-Charlier distribution with nonlinear correlations (including higher-order cross products of \( w' \) and \( s' \)) instead of the Gaussian model with a simple linear correlation. The authors showed that observed variations of the coefficient \( b_0 \) from 0.4 to 0.63 can be explained by the nonlinear JFD model, including up to fourth-order cross products. The systematic dependence of \( b_0 \) on the nonlinearity of the JFD is basically in agreement with the results of the present paper. However, Milne et al. [1999] give no information on the dependence of the cross products on stability and other environmental conditions. Therefore a direct comparison of the results is not possible.

5. Conclusions

In the present study, the coefficient \( b \) of the REA method was investigated by simulation experiments using eddy correlation measurements of three different scalar quantities. The wide range of stability conditions during the field campaign allowed to describe \( b \) as a function of the dimensionless stability parameter \( z/L \) within the framework of Monin-Obukhov similarity theory. In good agreement with previous observations over various surface types, a constant average value of 0.56 ± 0.01 was obtained for unstable and near-neutral conditions for all scalar quantities with no dead band applied. For stable conditions, however, a significant increase with stability was observed up to values of about 0.63, which could be well described by a logarithmic functional relationship between \( b_0 \) and \( z/L \). The present relationship is in better agreement with most results reported in the literature (especially for unstable and near-neutral conditions) than the relationship proposed by Andreas et al. [1998].

The influence from environmental controlling parameters other than \( z/L \) could be excluded, because the deviations from the fitted similarity relationship showed a purely random characteristic. The findings also indicate that an average similarity relationship for \( b \) might be more appropriate for REA flux calculations than the use of individual values from simultaneous EC measurements of sensible heat. Nevertheless, it is highly recommendable for each field application to investigate the individual \( b \) values in order to check the validity of the parameterization or to detect measurement problems and failures.

The influence of a dead band on the coefficient \( b \) was analyzed for various stability conditions, and numerical relationships between \( b \) and the normalized dead-band width were determined. It was shown that previously published parameterization schemes [Businger and Oncley, 1990; Patey et al., 1993] mainly reflected the limited stability ranges covered by the respective database and that they can be reconciled if the systematic stability dependence is considered. The present findings support the suggestion of Wesely and Hart [1994] to apply a normalized dead band of about 0.7 in order to have an increased concentration difference between updrafts and downdrafts combined with a minimal variability of the coefficient \( b \).

The presented results and considerations elucidate that the systematic variation of \( b \) with \( z/L \) is related to the nonlinearity in the joint frequency distribution of the vertical wind speed and the scalar quantity. Further investigation should focus on the explanation of this nonlinearity which may be influenced by the different integral timescales of the vertical wind speed and scalar fluctuations, depending on stability as well as on the boundary layer height.

Acknowledgments. The present study was supported by the European Union through the project EXAMINE (EHSV-CT94-0426) and by the Max Planck Society. We would like to thank Paul Gärtner, the local farmer in Bellheim, for allowing us to perform the experiment on his field and for his valuable support. The first author is indebted to Atsumu Ohmura, Ray Desjardins, and Matthias Rotach for reviewing the Ph.D. thesis, on which this paper is based.

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