系所別:　　水文科學研究所　　　科目:　　　　應用數學

1. [10%] Find the 2×2 matrix $[A]$ such that $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \times [A] = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$.

2. [10%] Find the value of a such that there exists non-trivial solution for the system: $\begin{cases} 2x-5y+3z=0 \\ -6x+ay+2z=0 \\ -4x-y+5z=0 \end{cases}$.

3. [10%] Find all functions $\phi(x,y)$ with $\phi_x = e^{xy} + xye^{xy} + \cos x + 1$, $\phi_y = x^2 e^{xy}$.

4. [20%] Solve the initial value problem: $\dfrac{dy}{dx} = \tan x \times \coth y$, $y(0)=0$.

5. [20%] Solve the initial value problem for $y = y(t)$: $\dfrac{d^2 y}{dt^2} - 2\dfrac{dy}{dt} = e^t$, $y(0)=0$, $\dfrac{dy}{dt}(0)=0$.

6. For the function $f(x,y,z) = x^2 y - 2x - y$, find

   (a) [5%] the unit vector pointing in the direction of steepest increase at $(3,1)$, and

   (b) [5%] the directional derivative at $(3,1)$ in the direction $\vec{u} = \vec{i} \cos\tfrac{1}{4}\pi + \vec{j} \sin\tfrac{1}{4}\pi$.

7. [10%] By use of the divergence theorem to evaluate the surface integral: $\iint\limits_S \vec{F} \cdot d\vec{A}$, where the vector $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S is the surface of a sphere $x^2 + y^2 + z^2 = 1$.

8. (a) [5%] Evaluate $\nabla \times \nabla \phi$ for $\phi = x^2 y^3 - 8$.

   (b) [5%] Evaluate $\nabla \cdot \nabla \times \vec{v}$ for $\vec{v} = -yz\vec{i} + z^2\vec{j} + xy\vec{k}$.