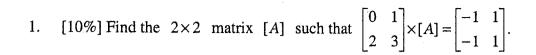
國立中央大學九十二學年度碩士班考試入學招生試題卷

系所別:

水文科學研究所

科目

應用數學





- 2. [10%] Find the value of a such that there exists non-trivial solution for the system: $\begin{cases} 2x 5y + 3z = 0 \\ -6x + ay + 2z = 0 \\ -4x y + 5z = 0 \end{cases}$
- 3. [10%] Find all functions $\phi(x, y)$ with $\phi_x = e^{xy} + xye^{xy} + \cos x + 1$, $\phi_y = x^2 e^{xy}$.
- 4. [20%] Solve the initial value problem: $\frac{dy}{dx} = \tan x \times \coth y$, y(0) = 0.
- 5. [20%] Solve the initial value problem for y = y(t): $\frac{d^2y}{dt^2} 2\frac{dy}{dt} = e^t$, y(0) = 0, $\frac{dy}{dt}(0) = 0$.
- 6. For the function $f(x, y, z) = x^2y 2x y$, find
 - (a) [5%] the unit vector pointing in the direction of steepest increase at (3,1), and
 - (b) [5%] the directional derivative at (3,1) in the direction $\vec{u} = \vec{i} \cos \frac{1}{4} \pi + \vec{j} \sin \frac{1}{4} \pi$.
- 7. [10%] By use of the divergence theorem to evaluate the surface integral: $\iint_S \vec{F} \cdot d\vec{A}$, where the vector $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S is the surface of a sphere $x^2 + y^2 + z^2 = 1$.
- 8. (a) [5%] Evaluate $\nabla \times \nabla \phi$ for $\phi = x^2 y^3 8$.
 - (b) [5%] Evaluate $\nabla \cdot \nabla \times \vec{v}$ for $\vec{v} = -yz\vec{i} + z^2\vec{j} + xy\vec{k}$.