所別：水文科學研究所碩士班　　科目：應用數學

1. Solve the following ordinary differential equations for $y = y(x)$:

(a) [10%] $e^x \frac{dy}{dx} = 2(x+1)y^2, \quad y(0) = \frac{1}{6}$

(b) [10%] $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0, \quad y(0) = 1, \quad y\left(\frac{\pi}{2}\right) = 0$

(c) [20%] $\frac{d^2y}{dx^2} - 4y = e^{-2x} - 2x, \quad y(0) = 0, \quad \frac{dy(0)}{dx} = 0$

2. For the given matrices: $\mathbf{A} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$,

evaluate:

(a) [10%] $\mathbf{A}^T \cdot \mathbf{B}$　(b) [10%] $\mathbf{C}^2$　(c) [20%] $\mathbf{D}^{-1}$

3. For any twice continuously differentiable scalar function f and vector function $\vec{u}$, verify the following identities:

(a) [10%] $\nabla \times (\nabla f) = 0$　(b) [10%] $\nabla \cdot (\nabla \times \vec{u}) = 0$