

國立中央大學99學年度碩士班考試入學試題卷

所別：水文與海洋科學研究所碩士班 不分組（一般生） 科目：應用數學 共 1 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷（卡）內作答

1. Solve $t \frac{dy}{dt} + y(t) + 4 = 0$ (15 points)

2. Solve the initial value problem (15 points)

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 2y = 0, \text{ at } t = 0 \dots y = 1, dy/dt = -1$$

3. Solve the following systems of linear equations (10 points)

$$5x + 3y - 3z = -1$$

$$3x + 2y - 2z = -1$$

$$2x - y + 2z = 8$$

4. The position vector of a moving particle as a function of time is denoted as $\vec{r}(t)$, where $t (\geq 0)$ is the time. For the following $\vec{r}(t)$, please describe the trajectory, the velocity vector, the speed, and the acceleration vector of the particle:

a. $\vec{r}(t) = \sin t \vec{j}$ (15 points)

b. $\vec{r}(t) = 10e^{-t} \vec{i} + 5e^{-t} \vec{j}$ (15 points)

5. A horizontal scalar field is given as $f(x, y) = 2x^2 + 3y^2$. This scalar field is in balance with a vector field $\vec{v}(x, y)$ as $\nabla f + \vec{k} \times \vec{v} = 0$, where \vec{k} is the unit vector in z direction.

a. find the gradient f at the point (2, 1) (5 points)

b. find the vector $\vec{v}(2, 1)$ (10 points)

c. find $\nabla \times \vec{v}$ at the point (2, 1) (10 points)

d. find $\nabla \cdot \vec{v}$ at the point (2, 1) (5 points)

參考用